Patent Licensing Contract Example: Hidden Characteristics

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Preliminaries


Suppose a patent owner with a cost-reducing innovation can profit only by licensing it to a manufacturer, and that there is a manufacturer who is a monopolist in her product market who can use the innovation in her production process.

Suppose the owner considers licensing contracts with a fixed payment (non-negative) $F$, and a (non-negative) royalty of $\epsilon$ per unit of production, so a contract is defined by $\{F, \epsilon\}$. 
Assumptions, notation

Buyer:

- total cost without innovation = $c^0 Q$
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- with innovation = $cQ$, $c < c^0$

Demand is $D(p)$

Profit (gross of fixed payment), as a function of average cost:

$$\pi(x) = \left[p(x) - x\right] D(p(x))$$

where $p(x) = \arg\max p - x$:$D(p)$

which has the necessary condition (MR = MC):

$$\left[p - x\right] D'(p) + D(p) = 0 \quad (1)$$
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Symmetric information problem

Simplify notation: let $D(x) = D(p(x))$

Seller:

$$\max \{ F + \epsilon D(c + \epsilon) \}$$

s.t.

$$\pi(c + \epsilon) - F \geq \pi(c^0)$$

$$\epsilon \geq 0$$

$$F \geq 0$$

(PC)
Solving symmetric information problem

First, it is clear that the (PC) binds. Suppose not, and assume seller chooses optimal $\epsilon^*$. Then seller can increase $F$ without violating (PC), and increasing $F$ increases the objective function, so cannot be at an optimum for $F$ until (PC) binds.

Now we can show that optimal royalty is $\epsilon = 0$. Since (PC) binds, $F = \pi(c + \epsilon) - \pi(c^0)$. Substitute into the objective function and do an unconstrained optimization (we’ll confirm the non-negativity constraints hold at the end).

$$
\max F + \epsilon D(p(c + \epsilon))
= \max \pi(c + \epsilon) - \pi(c^0) + \epsilon D(p(c + \epsilon))
= \max [p(c + \epsilon) - (c + \epsilon)D(p(c + \epsilon)) - \pi(c^0) + \epsilon D(p(c + \epsilon))
= \max [p(c + \epsilon) - c]D(p(c + \epsilon)) - \pi(c^0)
$$
Solving symmetric (cont.)

The necessary condition is

\[ \frac{\partial p}{\partial \epsilon} D(p) + p \frac{\partial D(p)}{\partial p} \frac{\partial p}{\partial \epsilon} \]

which we can rearrange as

\[ \frac{\partial p}{\partial \epsilon} \left[ D(p) + p \frac{\partial D(p)}{\partial p} \right] \]

but from (1) the term in square brackets is equal to \((c + \epsilon)D'(p)\) (MR = MC), which is negative (since demand curves slope down). Therefore, the seller should set \(\epsilon\) as small as possible: given the non-negativity constraint, \(\epsilon = 0\).

Finally, with \(\epsilon = 0\), we have \(F = \pi(c) - \pi(c^0)\). Since \(c < c^0\), and it is easy to show (and it is intuitively obvious) that profit is decreasing in cost, \(F > 0\), which satisfies its non-negativity constraint.
Set the royalty ($\epsilon$) to zero, and set the fixed payment ($F$) to extract all of the buyer’s surplus from using the innovation (all of the incremental profit, $F = \pi(c) - \pi(c^0)$).

(The buyer’s surplus is the incremental downstream profit from using the lower cost production process.)
Consider a special case: the new production cost can be either Good or Bad (though both are better than the old cost):

\[ c^G < c^B < c^0 \]

The optimal symmetric information contract can now be expressed as two contracts, one of which will be offered by the principal depending on which cost the innovation delivers:

Good: \( \{ \epsilon^{G*} = 0, F^{G*} = \pi(c^G) - \pi(c^0) \} \) \hspace{1cm} (2)

Bad: \( \{ \epsilon^{B*} = 0, F^{B*} = \pi(c^B) - \pi(c^0) \} \) \hspace{1cm} (3)
Asymmetric information problem

Now suppose the buyer has private information: only she knows for sure which level of cost the innovation yields in her production process.

The seller has a prior belief that there is a probability \( q \) that the cost will be \( c^G \).
Seller’s problem

\[
\begin{align*}
\max_{\{F^G, \epsilon^G, F^B, \epsilon^B\}} & \quad q \left[ F^G + \epsilon^G D(c^G + \epsilon^G) \right] + (1-q) \left[ F^B + \epsilon^B D(c^B + \epsilon^B) \right] \\
\text{s.t.} & \quad \pi(c^G + \epsilon^G) - F^G - \left[ \pi(c^G + \epsilon^B) - F^B \right] \geq 0 \quad (\text{IC-G;} \ \mu) \\
& \quad \pi(c^B + \epsilon^B) - F^B - \left[ \pi(c^B + \epsilon^G) - F^G \right] \geq 0 \quad (\text{IC-B;} \ \lambda) \\
& \quad \pi(c^G + \epsilon^G) - F^G - \pi(c^0) \geq 0 \quad (\text{PC-G;} \ \rho) \\
& \quad \pi(c^B + \epsilon^B) - F^G - \pi(c^0) \geq 0 \quad (\text{PC-B;} \ \delta) \\
& \quad \left\{ F^G, F^B, \epsilon^G, \epsilon^B \right\} \geq 0 \quad (\alpha^G, \alpha^B, \beta^G, \beta^B)
\end{align*}
\]
Step 1

We can show that (PC-G) does not bind (so $\rho = 0$):

\begin{align*}
\pi^B &\geq \pi^0 \quad \text{(from (PC-B))} \\
\pi^G &\geq \pi(c^G + \epsilon^B) - F^B \quad \text{(from (IC-G))} \\
\pi(c^G + \epsilon^B) - F^B &> \pi(c^B + \epsilon^B) - F^B \quad \text{(by $\frac{\partial \pi}{\partial c} < 0$)}
\end{align*}

So, $\pi^G > \pi^B \geq \pi^0 \implies \rho = 0$, and (PC-G) binds.
Step 2

\[
\frac{\partial L}{\partial F^G} : q - \mu + \lambda + \alpha^G = 0 \iff \mu = q + \lambda + \alpha^G > 0 \quad (4)
\]

because \( q > 0, \{\lambda, \alpha^G\} \geq 0 \). (4) implies that (IC-G) is binding, so:

\[
F^G = \pi(c^G + \epsilon^G) - \pi(c^G + \epsilon^B) + F^B \quad (5)
\]
Step 3

\[
\frac{\partial \mathcal{L}}{\partial F_B} : (1-q) + \mu - \lambda - \delta + \alpha^B = 0 \iff 1 + \alpha^G + \alpha^B = \delta > 0 \tag{6}
\]

By using (4), and the fact that \( \{\alpha^G, \alpha^B\} \geq 0 \). \( \delta > 0 \) means (PC-B) is binding, so:

\[
F^B = \pi(c^B + \epsilon^B) - \pi(c^0) \tag{7}
\]

Then, by substituting (7) into (5):

\[
F^G = \pi(c^G + \epsilon^G) - \pi(c^G + \epsilon^B) + \pi(c^B, \epsilon^B) - \pi(c^0) \tag{8}
\]
Step 4a

First, we’re going to need to know by how much changing the royalty ($\epsilon$) changes profits:

$$\pi(c + \epsilon) = [p(c + \epsilon) - (c + \epsilon)] D(p(c + \epsilon))$$

$$\frac{\partial \pi}{\partial \epsilon} = \left( \frac{\partial p}{\partial \epsilon} - 1 \right) D + (p - (c + \epsilon)) \frac{\partial D}{\partial p} \frac{\partial p}{\partial \epsilon}$$

$$= \frac{\partial p}{\partial \epsilon} \left[ D + (p - (c + \epsilon)D') \right] - D \quad (9)$$

But substituting the first-order condition for optimal pricing in (1), we have:

$$\frac{\partial \pi}{\partial \epsilon} = -D(p(c + \epsilon)) \quad (10)$$
Step 4b

Using (10), we have $\partial L/\partial \epsilon^G$:

$$qD(c^G + \epsilon^G) + q\epsilon^G D'(c^G + \epsilon^G) - \mu D(c^G + \epsilon^G) + \lambda D(c^B + \epsilon^G) + \beta^G = 0$$

From (4) we have $q - \mu + \lambda = -\alpha^G$, so

$$(q - \mu + \lambda)D(c^G + \epsilon^G) = -\alpha^G D(c^G + \epsilon^G)$$

and substituting into $\partial L/\partial \epsilon^G$ we get

$$\beta^G = -q\epsilon^G D'(c^G + \epsilon^G) + \alpha^G D(c^G + \epsilon^G) + \lambda \left[ D(c^G + \epsilon^G) - D(c^B + \epsilon^G) \right]$$
Step 4c

Using this result for $\beta^G$ we can show that $\epsilon^G = 0$ (no royalty in Good contract):

$$\beta^G = -q\epsilon^G D'(c^G + \epsilon^G) + \alpha^G D(c^G + \epsilon^G) + \lambda \left[ D(c^G + \epsilon^G) - D(c^B + \epsilon^G) \right]$$

If $\beta^G > 0$, then $\epsilon^G = 0$ by KT conditions.

If $\beta^G = 0$: We know $-qD' > 0$, $D > 0$, and $D(c^G + \epsilon^G) - D(c^B, \epsilon^G) > 0$ (the latter because optimal price is increasing in cost, so equilibrium output is decreasing in cost, making $D(c^B) < D(c^G)$). Since each term is non-negative and all of the multipliers are non-negative, it must be that

$$\epsilon^G = \alpha^G = \lambda = 0$$
Result 1

Result

The *Good* contract royalty is zero, \( \epsilon^G = 0 \).

This is generally true for hidden characteristics contracts, and is sometimes referred to as “No distortion at the top”.
Step 5a

\[
\frac{\partial L}{\partial \epsilon_B} : (1 - q)D(c^B + \epsilon^B) + (1 - q)\epsilon^B D'(c^B + \epsilon^B) + \mu D(c^G + \epsilon^B) \\
- \lambda D(c^B + \epsilon^B) - \delta D(c^B + \epsilon^B) + \beta^B = 0 \tag{11}
\]

Using the same trick we used with \(\frac{\partial L}{\partial \epsilon^G}\), but substituting this time from (6), we get:

\[
\mu \left[ D(c^G + \epsilon^B) - D(c^B + \epsilon^B) \right] - \alpha^B D(c^B + \epsilon^B) + \\
(1 - q)\epsilon^B D'(c^B + \epsilon^B) + \beta^B = 0 \tag{12}
\]
Step 5b

Using this result:

\[ \mu \left[ D(c^G + \epsilon^B) - D(c^B + \epsilon^B) \right] - \alpha^B D(c^B + \epsilon^B) + \\
(1 - q)\epsilon^B D'(c^B + \epsilon^B) + \beta^B = 0 \quad (13) \]

Suppose \( \epsilon^B = 0 \). From (7)

\[ F^B = \pi(c^B) - \pi(c^0) > 0 \]

which implies that \( \alpha^B = 0 \) (since the non-negativity constraint on \( F^B \) is not binding). Then, using (13) we have

\[ \mu \left[ D(c^G + \epsilon^B) - D(c^B + \epsilon^B) \right] + \beta^B = 0 \]

but this is a contradiction since \( \mu > 0 \) by (4) and we know \( \beta^B \geq 0 \).

Therefore, \( \epsilon^B > 0 \).
The bad contract royalty is positive, $\epsilon^B > 0$.

This is generally true for hidden characteristics contracts, and is sometimes referred to as “Distortion at the bottom”.
Result 3

Result

The Good contract up-front payment is higher than for the Bad contract: \( F^G > F^B \).

Proof.

From (8), \( F^G = \pi(c^G) - \pi(c^G + \epsilon^B) + \pi(c^B + \epsilon^B) - \pi(c^0) \).

From (7), \( F^B = \pi(c^B + \epsilon^B) - \pi(c^0) \).

Since \( \epsilon^B > 0 \), \( \pi(c^G) > \pi(c^G + \epsilon^B) \iff F^G > F^B \).
Result 4

Result

The up-front payment for the Good contract when there is asymmetric information is less than when information is symmetric, $F^G < F^{G*}$.

Proof.

$F^{G*} = \pi(c^G) - \pi(c^0)$, so

$$F^{G*} - F^G = \pi(c^G + \epsilon^B) - \pi(c^B + \epsilon^B) > 0$$

because profits are decreasing in unit cost.
Result 5

Result

The up-front payment for the Bad contract when there is asymmetric information is less than when information is symmetric, $F^B < F^{B*}$.

Proof.

$$F^B = \pi(c^B + \epsilon^B) - \pi(c) < \pi(c^B) - \pi(c^0) = F^{B*}$$ because profits are decreasing in unit cost.
Summary

Here’s what we found about the optimal (asymmetric information) patent licensing contract for this problem:

1. The optimal contracts are separating: The Good type selects \( \{F^G, 0\} \), while the Bad type selects \( \{F^B, \epsilon^B\} \) (see Results 1, 2).

2. \( F^G > F^B \) (see Result 3).

3. \( \delta = 0 \Rightarrow PC-B \) is binding (see Step 3): No information rent at the bottom.

4. \( \rho = 0 \Rightarrow PC-G \) is not binding (see Step 1): Information rent at the top (also seen from \( F^G < F^G^* \), Result 4).

5. \( \epsilon^B > 0 \Rightarrow \) efficiency distorted at bottom (to make it less attractive to the Good type to masquerade as a Bad type, so that we can get more rent out of the more productive type) (see Result 2).

6. \( \epsilon^G = 0 \Rightarrow \) efficient at the top (don’t want to distort the most productive) (see Result 1).
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Contracts

Figure: The asymmetric information contract parameters