HIDDEN CHARACTERISTICS III: SIGNALING

SI 680, ICD: SIGNALING AND CONTRACTING
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1. What is a signal?

Bruce the Bold: Kingdom ruled by old king with a son but no daughters.[1] Wanted his only son to marry a wise princess in order that the kingdom be wisely managed when he passed away. (The son spent most of his time watching jousting matches.) Many princesses wanted to marry the prince so they could rule the kingdom. Of course, each claimed that she was wise. Therefore, the king proposed a test: they had to use the Lagrangean method to solve 50 constrained optimization problems. Only Suzy SmartyPants bothered to take the test; the rest knew that they would first have to go to college to learn how, and the kingdom (not to mention the pimply prince) were not worth that much.

Why is this test a good signal? Suzy will not actually need to solve an Lagrangean problems to manage the kingdom. But people who learned how to do that are smart enough to manage the kingdom. The others could try to pretend they are that smart, but it is too costly for them to be worth it.

2. Canonical signaling problem

The feature that distinguishes a signaling problem from our prior hidden characteristics problems (sometimes referred to as "adverse selection" problems) is that the agent is able to obtain and transmit a signal before the principal offers a contract. As usual, it is helpful to look at the timing of information and actions; by comparing the timing for adverse selection and signaling we can easily see the difference (Figure 2).

3. Education as signal

The seminal paper on signaling is A. Michael Spence. Job market signaling. *Quarterly Journal of Economics*, 87:355–74, 1973, for which he won the Nobel Prize in Economics. Below I present a version of his model. Spence presents a caricature

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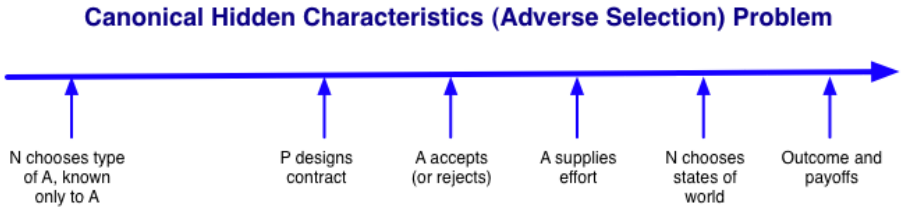
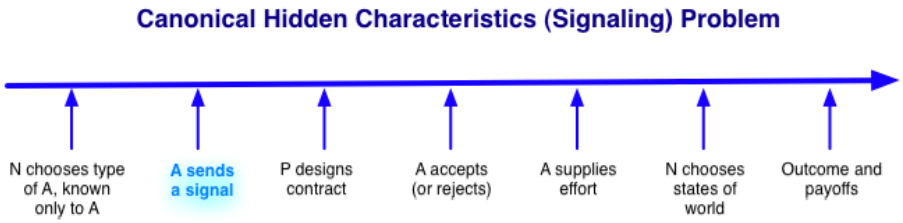
[1] An example like this is standard throughout the signaling literature, but this particular fanciful formulation I borrowed from Macho-Stadler and Perez-Castrillo [2001].





higher education as an example; please don't believe that he truly is claiming that education does not actually provide any useful knowledge or skills![2]

Assume there are two types of user interface designers: Fast learners and Slow; the fraction ρ of the population is type F.

Designers get utility: $u_i = t(e) - c(e)$, where e is amount of completed education, c is cost of getting educated, t is transfer (may or may not depend on education: that depends on the contract offered by principal).

Cost of education: $c_F(e) < c_S(e)$ (to get the same degree, it takes the S types more hours of studying, more semesters, more tutoring).

The value to the firm of the different types of workers: $q_F > q_S$. To simplify the example, productivity is unaffected by education (!!).

[2]Spence is pretty serious about higher education: after being a Professor of Economics at Harvard, he became Dean of Arts and Sciences there, and later was Dean of the Business School at Stanford. While at Harvard, he taught both Bill Gates and Steve Ballmer in a graduate economic theory class; both received an A.

For this example, I will use the following specific values:

$$q_S = q \tag{1}$$

$$q_F = 2q \tag{2}$$

$$c_F(e) = \frac{e^2}{2} \tag{3}$$

$$c_S(e) = \frac{3e^2}{4} \tag{4}$$

We further assume that

- Firms have beliefs about whether UI applicants are F or S
- Education is observable, so firms can offer a transfer (contractual payment) that depends on education
- UI designers choose their education level to get the highest expected utility (taking into account the equilibrium wages for different levels of education).
- Firms design contracts to maximize their expected profits.
- The firms are in a competitive market for attracting employees.

Because firms will receive information before they write and offer contracts, we will need to take into account the updating of their beliefs. We will assume that firms do correct Bayesian updating. More strongly, we will assume that in market equilibrium, firms have *correct beliefs* about the qualities of workers with different amounts of education (different signals).[3]

In this problem, with only two types of workers, there are two possible contract forms in equilibrium:

- Pooling: all designers get same e, same t, firms correctly expect fraction ρ of F types.
- Separating: F-types get more e, earn higher t, firm correctly expects those with more e are F.

4. Solving for the pooling equilibrium contract

Under what conditions will signaling fail to separate?

If the firm believes all designers get the same education, and expects fraction ρ are F, then average productivity is

$$\rho(2q) + (1-\rho)q = (1+\rho)q$$

[3]You can think of correct beliefs as the consequence of experience over time, disciplined by competition in the market: if a firm systematically is either over-optimistic or under-optimistic about worker quality, it will lose money and go out of business in a competitive market.

If the firm is in competitive market to get workers, it must pay them their full marginal value. Suppose the firm offers two contracts:

- $t = (1+\rho)q$ if $e \geq SCREEN$, where $SCREEN$ is some threshold level of education
- $t = q$ if $e \leq SCREEN$ (if a designer shows up with no education, treat as Slow)

How much education (e) will designers choose to get? Certainly, no one gets $e > SCREEN$ (that is, strictly more than $SCREEN$): **Q: Why not? Because it doesn't increase wage, and it costs something to obtain.**

We can find the optimal contract level of e by analyzing the incentive compatibility constraints (IC).

- F chooses $SCREEN$ if $(1+\rho)q - \frac{1}{2}SCREEN^2 > q$
- S choooses $SCREEN$ if $(1+\rho)q - \frac{3}{4}SCREEN^2 > q$

Clearly only the S condition is binding. It will hold if $\sqrt{(4/3)\rho q} > SCREEN$.

Example 1. Let $\rho = 1/3$, $q = 9$. Then, there will be a pooling equilibrium if $2 > SCREEN \geq 0$, with $t = 12$ for all designers.

Remark 1. Any level of e between 0 and 2 is an equilibrium.

Remark 2. But e is unproductive, so 0 is the most efficient equilibrium. (Sometimes, though not always, it is reasonable to expect market competition to lead to the most efficient signaling contract.)

5. Solving for the separating equilibrium contracts

So, is it possible to use the signal to separate the F from the S?

Suppose that for some $SCREEN$, the firm believes F types get $e \geq SCREEN$ and the S types get $e < SCREEN$. Then, the firm, competing for designers, offers the following transfers (wages):

$$t_F = 2q \text{ if } e \geq SCREEN \tag{5}$$

$$t_S = q \text{ if } e < SCREEN \tag{6}$$

Designers then choose how much education to get: $e \in \{0, SCREEN\}$. Their optimization gives us the incentive compatibility conditions:

$$2q - \frac{1}{2}SCREEN^2 > q \tag{F}$$

$$q > 2q - 3\frac{3}{4}SCREEN^2 \tag{S}$$

Or, combining these inequalities,

$$\frac{3}{2}SCREEN^2 > q > \frac{1}{2}SCREEN^2$$

Example 2. $q = 9 \Rightarrow 2.5 < SCREEN < 4.2$.

Remark 3. $SCREEN$ higher than in pooling: necessary to discourage S types.

Remark 4. But not so high that it wastes all surplus for F types.

Remark 5. But, there is dissipation: investment in unproductive e.

6. Signaling summary

(1) Necessary condition for separating equilibrium: cost of signaling higher for low quality.
   - This is what enables high quality types to distinguish themselves: they can better afford to be wasteful!
   - **Crucial point:** Signal only works if it doesn't make sense for low quality type to give the high quality signal.

(2) Signaling is dissipative: truth comes with a cost in wasteful signaling.
(3) Often the same social welfare can be achieved with less signaling.
(4) Sometimes everyone invests in signaling but low and high quality not distinguished.