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SI 563 Lecture 3

Solving Extensive Form Games: SPNE

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Fall 2008

Agenda

– **Backward induction (First Half)**

–**Subgame perfection (Second Half)**

Solving Extensive-Form Games

Look Forward, Reason Back

(Watson Chapter 14, 15)

Comparison of Normal and Extensive Forms

- An extensive form can be translated into a normal form
- Normal-form concepts are valid for every game
- Nash equilibrium is still a valid solution concept for an extensive form
- In some games, the extensive form most precisely captures the order of moves and the information structure

Example: Entry and predation.
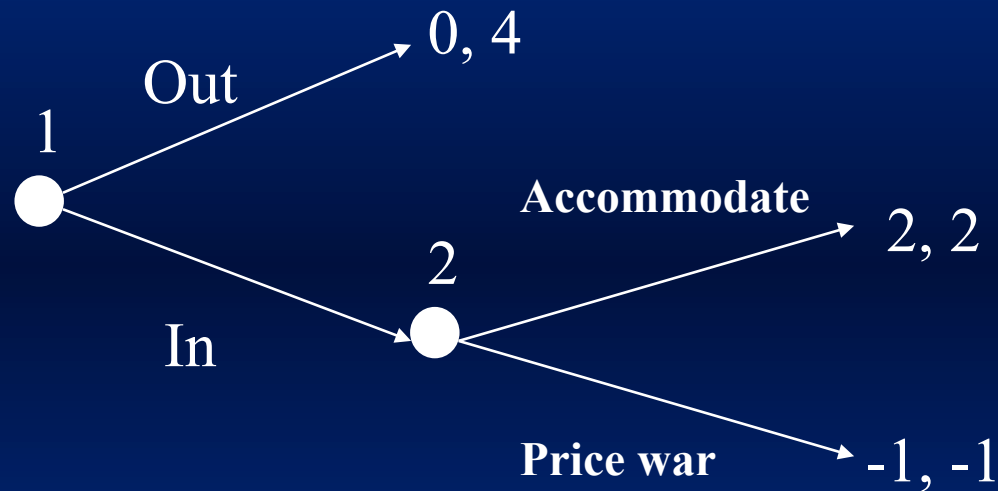
Player 1: potential entrant
Player 2: incumbent

Example: Normal Form for the Entry and Predation Game

	A	P
I	**2,2**	**-1,-1**
O	**0,4**	**0,4**

Two pure strategy Nash equilibria:
(I, A) and (O, P)
However, (O, P) is not plausible.
Note: if eliminate weakly dominated strategy, problem!

Sequential Rationality

- **An optimal strategy for a player should maximize his or her expected payoff, conditional on every information set at which this player has the move**

- **Player i's strategy should specify an optimal action from each of player i's information sets, even those that player i does not believe (ex ante) will be reached in the game**

Backward Induction

- **If sequential rationality is common knowledge, each player should *look ahead* to consider what others will do in the future in response to her move at a particular information set**

- **Backward induction: the process of analyzing a game from back to front (from information sets at the end of the tree to those at the beginning). At each information set, strike out actions that are dominated, given the terminal nodes that can be reached.**
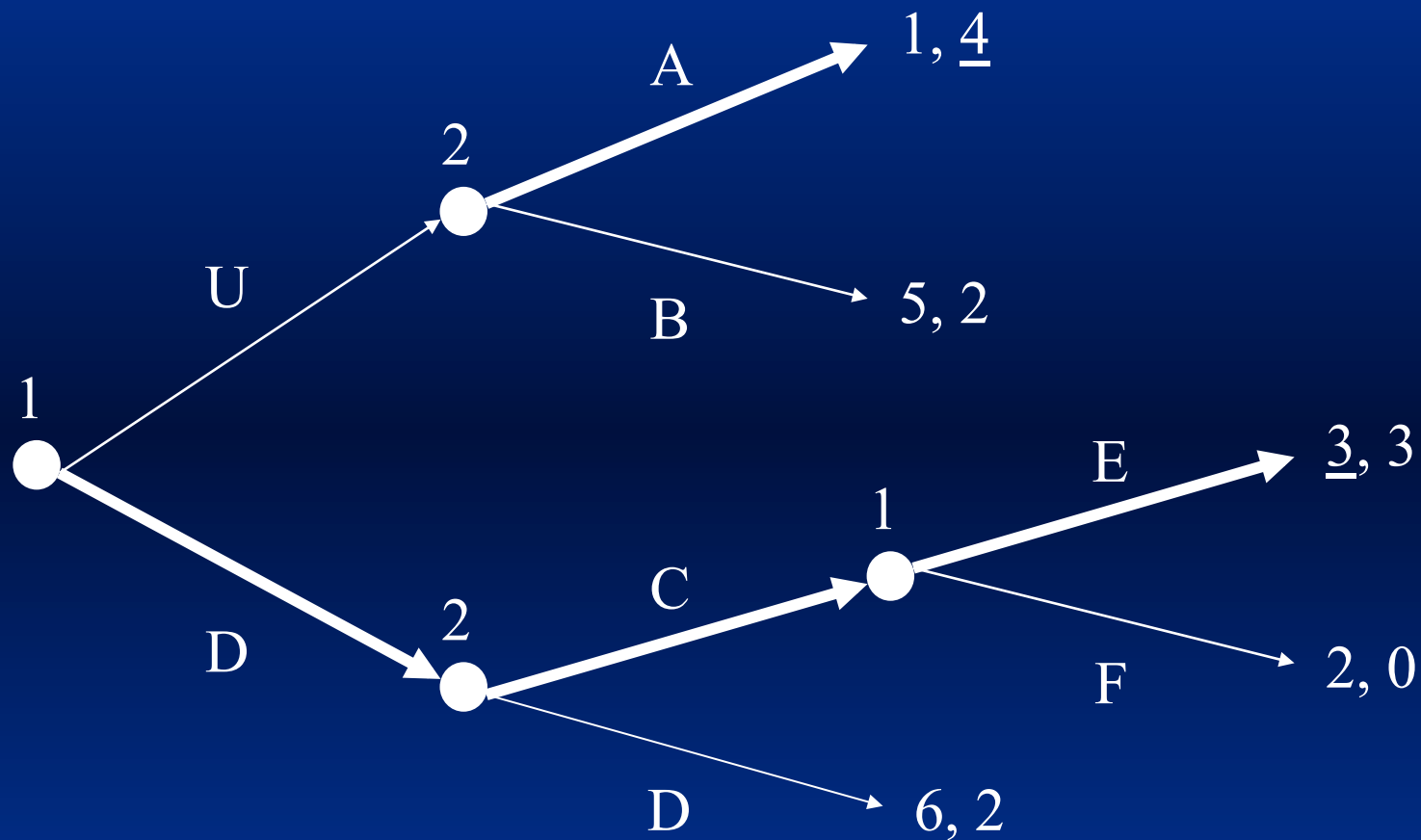
Example: Charlie Brown vs. Lucy

Lucy's preference (best to worst):
        CB falls down
        Nothing happens
        CB kicks the ball

Charlie Brown's preferences: opposite

Diagram courtesy: Dr. Tayfun Sönmez
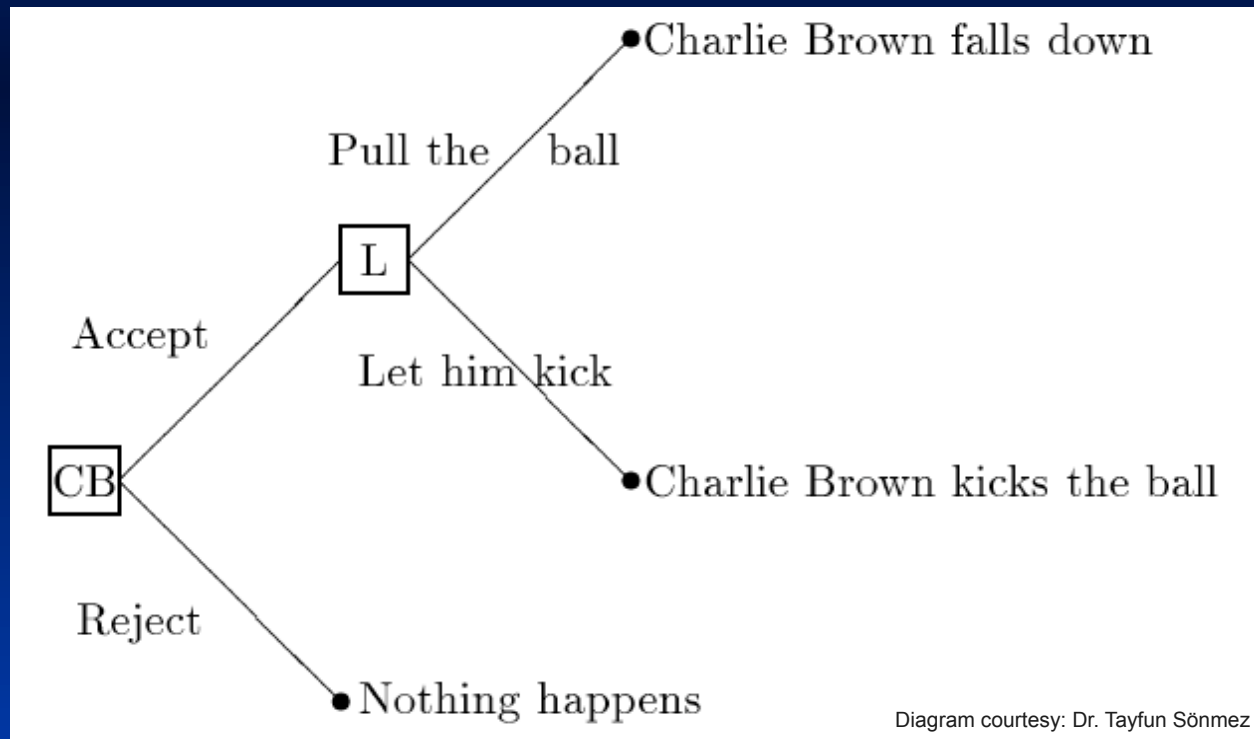
Example: Backward induction

A single sequentially rational strategy profile:
(DE, AC), check that this is also a Nash equilibrium

Example: Voting for a Payraise

Three legislators are voting on whether to give themselves a pay raise.
All three want the pay raise; however,
Each face a small cost in voter resentment c>0.
The benefit for the raise is greater than cost: b>c
They vote in the order 1-2-3. Simple majority rule
What is the outcome obtained by backward induction? (N; NY; nyyn)
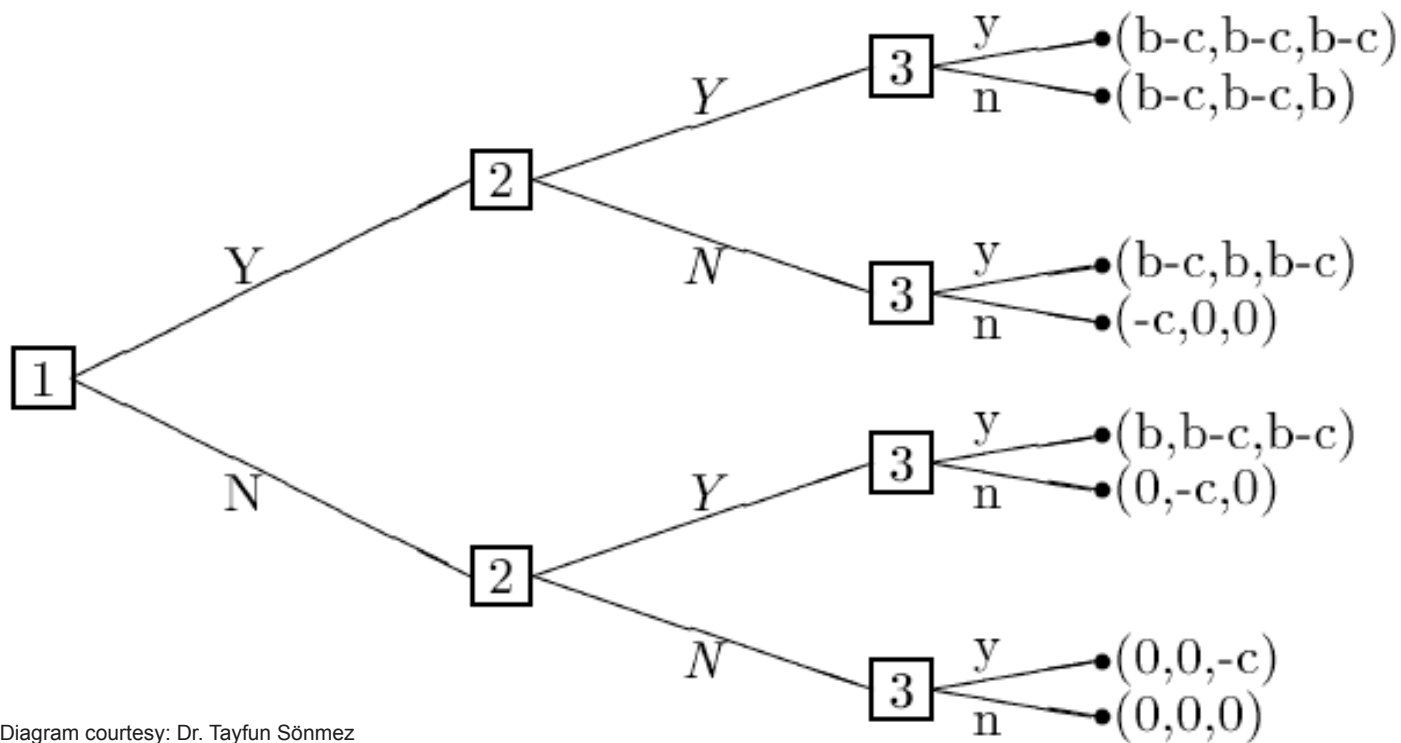
Diagram courtesy: Dr. Tayfun Sönmez

Advertising and Entry

- **An established retailer is facing possible competition from a rival**

- **The established retailer can try to stave off entry by engaging in a costly advertising and price cutting campaign**

Speed and Flexibility

- **The rival is fast and flexible, so its policy is to wait and decide at the last possible instant its entry choice**

- **Thus, the rival observes the initiation of this campaign before making its entry decision**

Strategies and Payoffs

- **Incumbent: Advertise or not**
- **Rival: Enter or not**

Payoffs:

- **Incumbent (Best to worst)**
  - **No entry and no ads**
  - **No entry and ads**
  - **Entry and no ads**
  - **Entry and ads**
- **Rival**
  - **Entry and no ads**
  - **No entry and ads**
  - **No entry and no ads**
  - **Entry and ads**

Game Tree

In — 1, 1

R

Ads — Out — 3, <u>3</u>

I

No ads

In — 2, <u>4</u>

R

Out — 4, 2
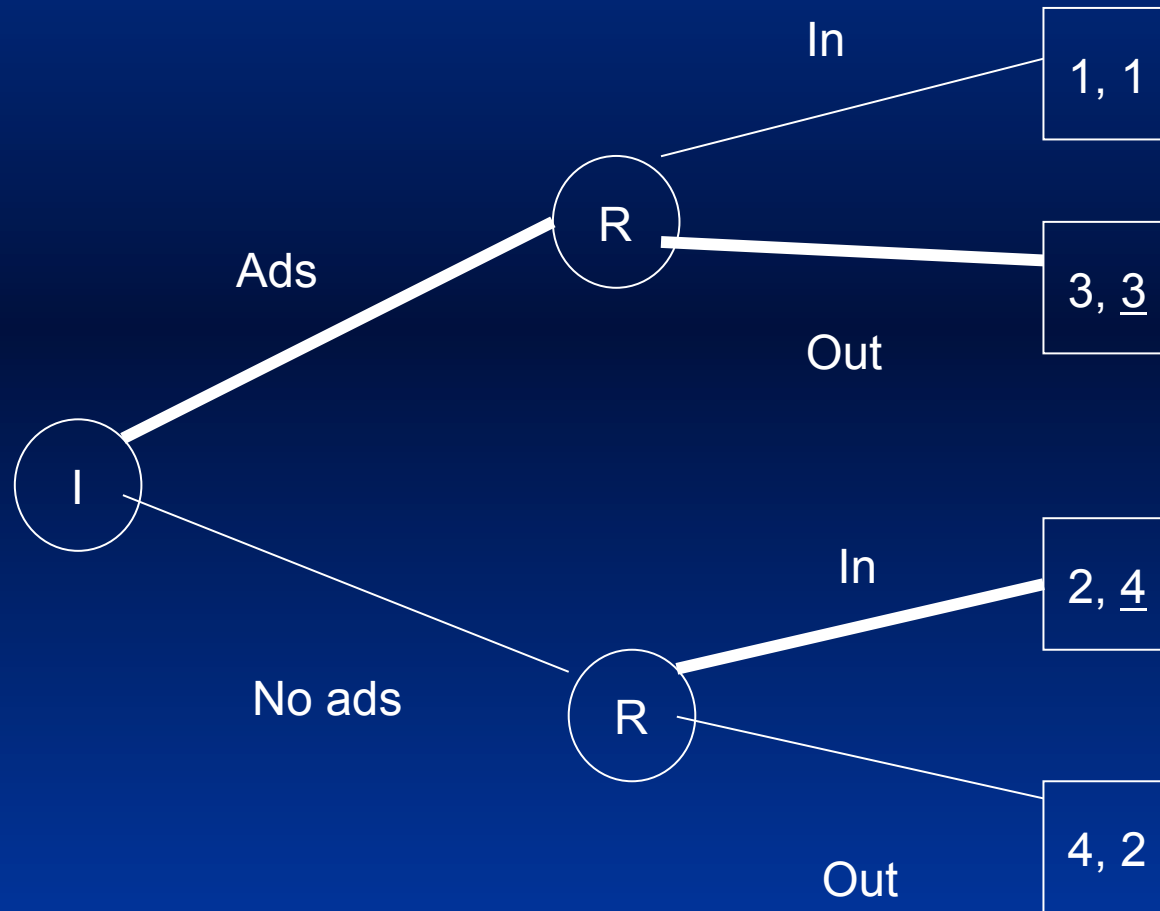
Equilibrium Prediction

- **The prediction from this model is that the incumbent will run its ad campaign and this will effectively forestall entry**

- **Notice that even in absence of actual entry, the *potential* competition from the rival eats into the incumbent's profits.**

Is Flexibility an Advantage?

- **Suppose that the rival is less flexible in its management practices.**

- **It must commit to enter or not *before* the advertising decision of the incumbent.**

- **How does this affect the outcome of the game?**

Game Tree – Stodgy Rival

```
                                          Ads          ┌──────┐
                                        ┌──────────────│ 1,1  │
                                        │              └──────┘
                               ┌───┐    │
                    In         │ I │════╪══════════════┌──────┐
              ╔═════════════════│   │    │              │ 4, 2 │
              ║                └───┘    No ads         └──────┘
         ┌───┐
         │ R │
         └───┘
              ╲                          Ads          ┌──────┐
               ╲                       ┌──────────────│ 3,3  │
                ╲   Out                │              └──────┘
                 ╲             ┌───┐   │
                  ╲════════════│ I │═══╪══════════════┌──────┐
                               │   │    │              │ 2, 4 │
                               └───┘   No ads         └──────┘
```
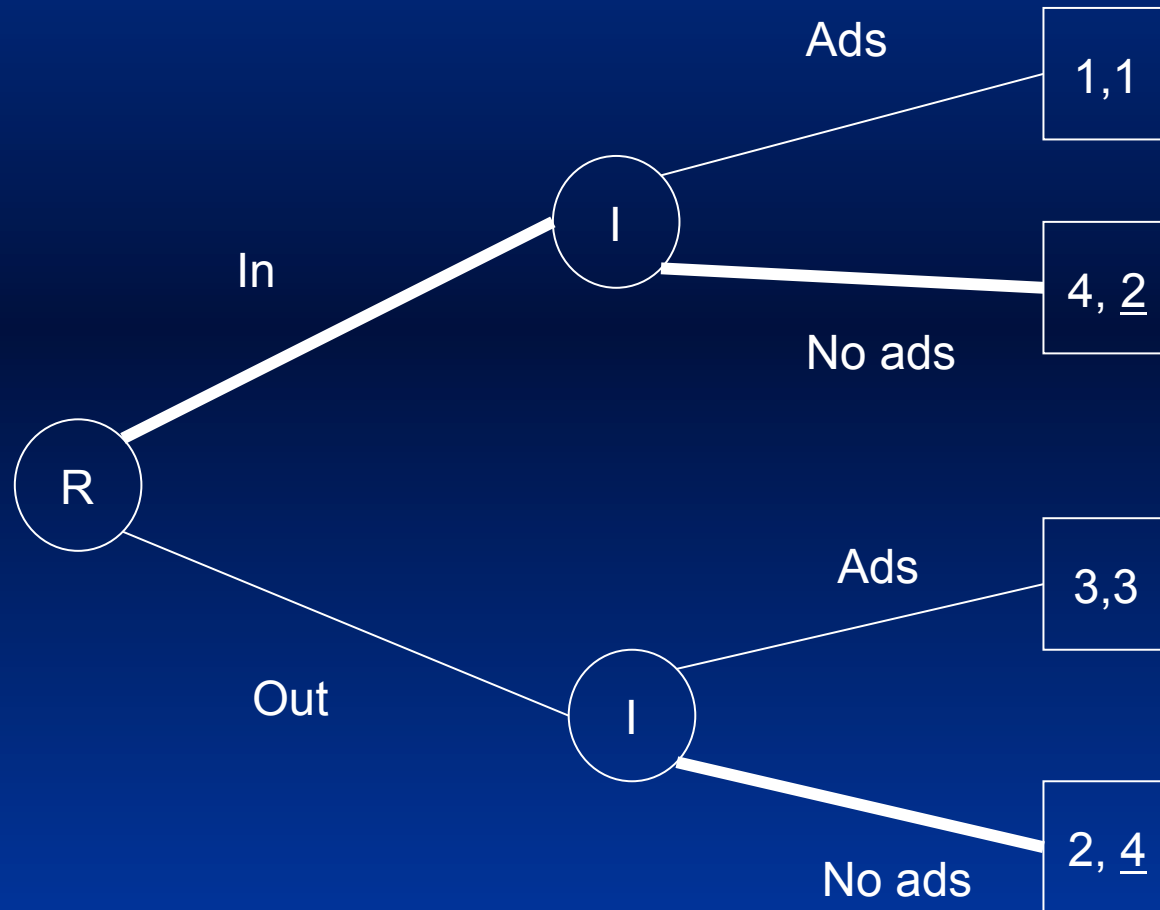
Equilibrium Prediction

- **Notice that now the prediction is that the rival will enter and the incumbent will not advertise**

- **The absence of flexibility on the part of the rival improves its outcome relative to the case where it retained flexibility.**

- **This game has a *first-mover advantage***

Do all games exhibit first-mover advantage?

- ## No

- ## Procurement contracts:
  - Two firms are bidding for a procurement contract, which will be awarded to the low bidder.
  - There is a cost to preparing a bid
  - Firm 1 chooses its bid followed by firm 2.
  - Clearly it pays to go second and undercut the bid of the first firm

Location Decisions

- **Three retailers (A, B, and C) are deciding their location decisions for an emerging metropolitan area**

- **Their decision is whether to locate in an urban mall or a suburban mall**

- **The urban mall has spots for 2 stores**

- **The suburban mall has spots for all 3.**

Motivation

- **The urban mall has more traffic than the suburban mall**

- **There are synergies in mall location**
  - **The presence of 2 or more large stores drives disproportionate traffic to that mall**

Strategies and Payoffs

- **Each retailer chooses where to locate: urban or suburban**
- **Payoffs (Best to worst)**
  - **Urban mall with other stores**
  - **Suburban mall with other stores**
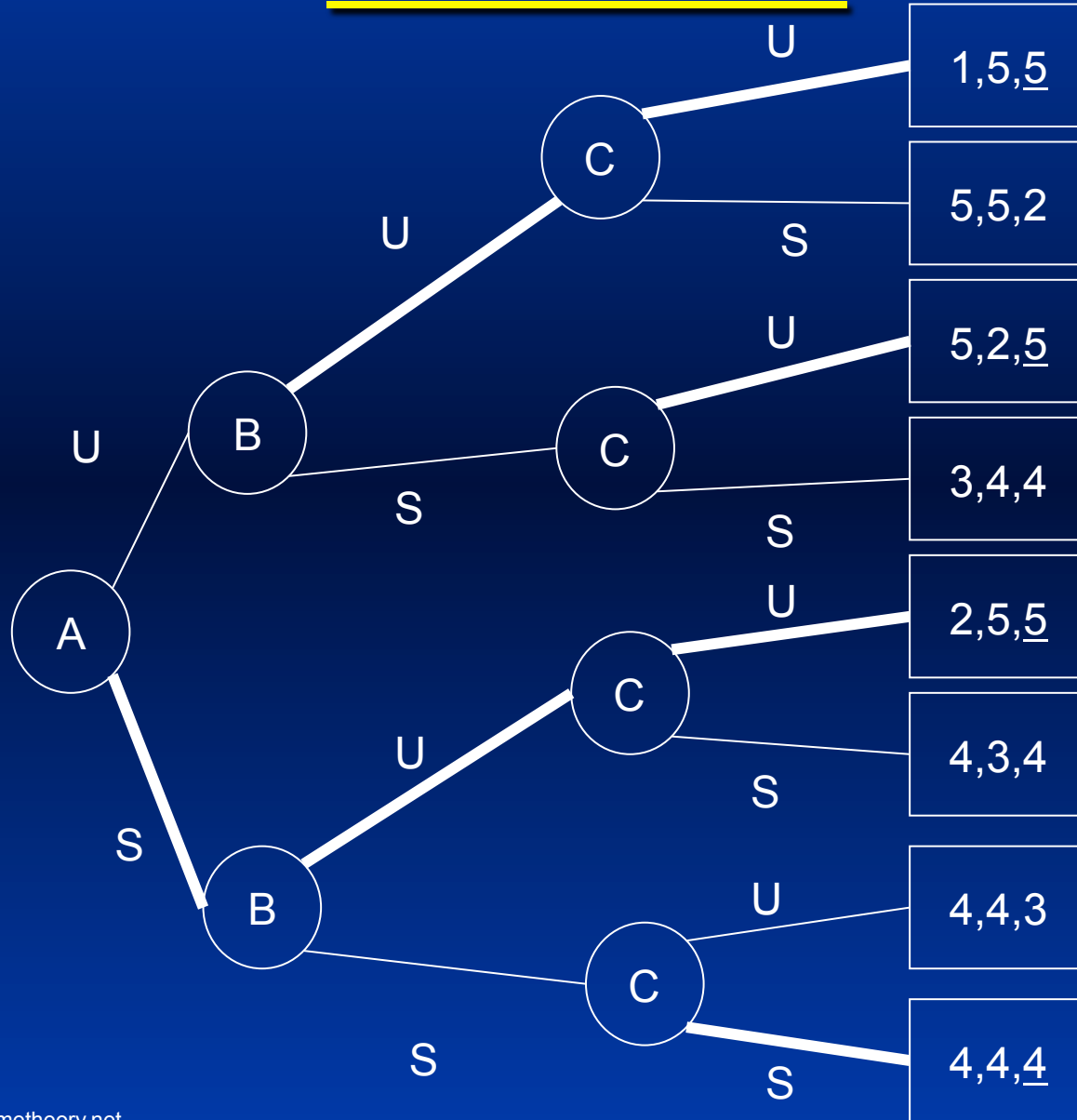 - **Urban mall alone**
  - **Suburban mall alone**
  - **No mall**

Mall Allocation

- **The malls are not strategic players.**
- **Urban: B and C have priority over A**
- **Suburban: Accept anyone**

<u>Timing</u>

- **Firm A moves first, followed by B, and then C**

Game Tree

Equilibrium Prediction

- **Firm C chooses suburban only if other 2 choose S**

- **Firm B knows that by choosing an urban location, C will follow suit, therefore it goes urban**

- **Firm A knows that B will go urban regardless and that C will follow B's lead, therefore if it goes urban, it will be shut out**

  – **Therefore A goes suburban and ends up alone**

General Principles

- **Sketch a game tree outlining who moves when**

- **Construct a ranking of outcomes for both you and your rivals.**

- **Look forward and reason back**

- **If the outcome is not a desirable one, think about how you might *change the game***

Ways to Change the Game

- **Contracting: Look ahead in thinking about the strategic implications of contract terms**

- **Change the order of moves (i.e. commitment) to gain a first-mover advantage.**

Potential Problems with Backward Induction: 3-Level Centipede

What is the outcome obtained by backward induction?

What are the Pareto efficient outcomes?

What do people actually do when they play the centipede game?
McKelvey and Palfrey (1992) *Econometrica*

Diagram courtesy: Dr. Tayfun Sönmez
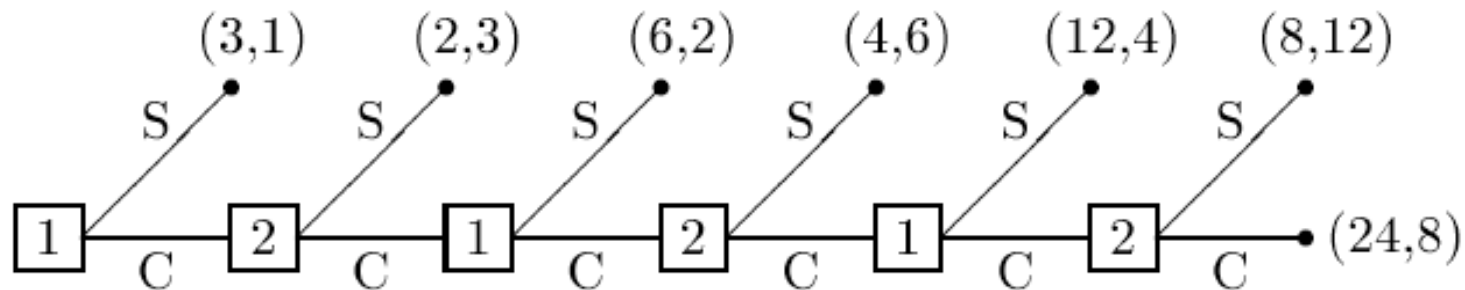
Another Problem with Backward Induction
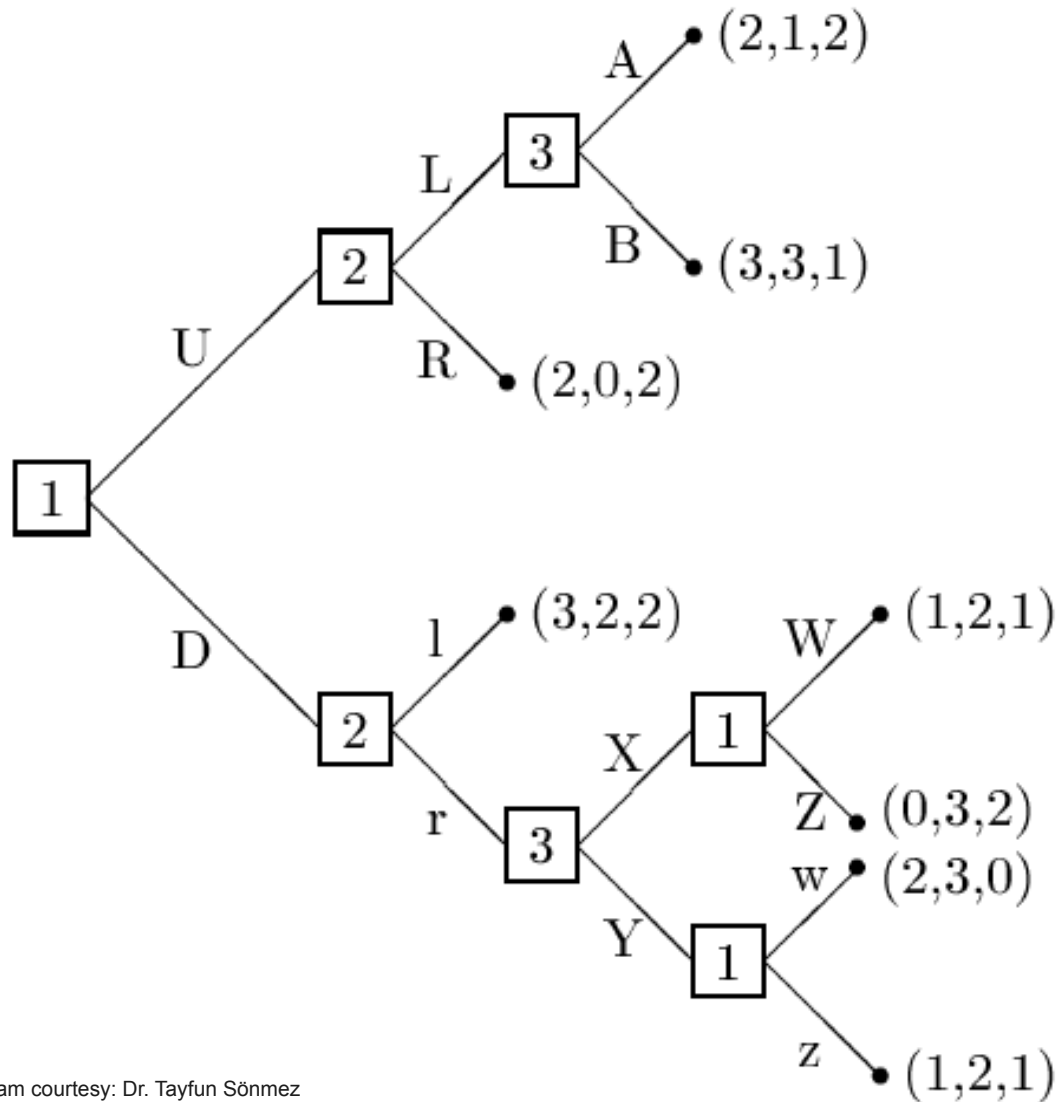

Diagram courtesy: Dr. Tayfun Sönmez

Solving Extensive-Form Games

Subgame Perfect Nash Equilibrium
(Watson Chapter 15)

Subgame and Proper Subgame

- **Given an extensive-form game, a node x in the tree is said to *initiate a subgame* if neither x (nor any of its successors) are in an information set that contains nodes that are not successors of x**

- **A *subgame* is the tree structure defined by such a node x and its successors**

- **Subgames that start from nodes other than the initial node are called *proper subgames***

Example: Subgames
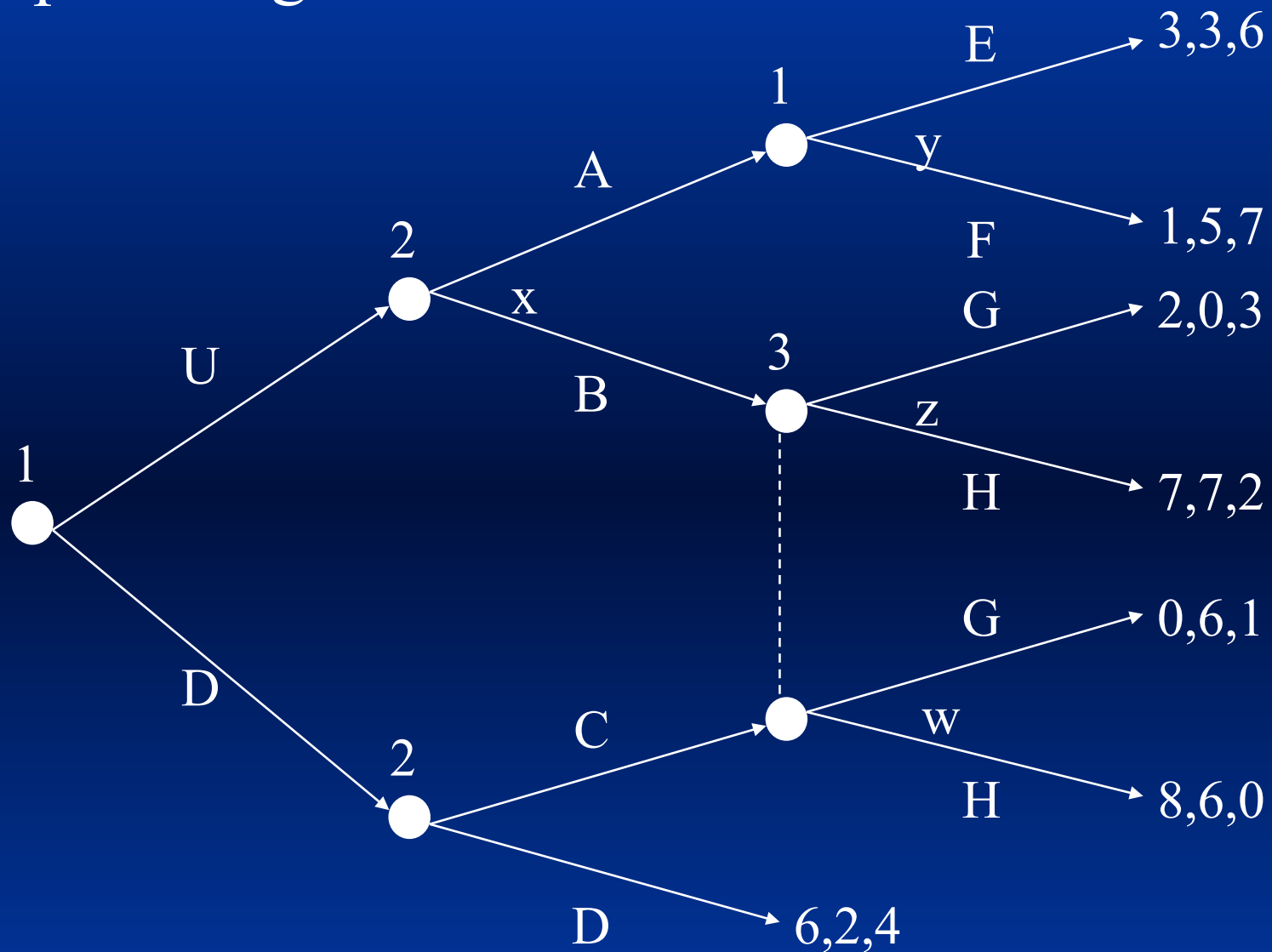
How many subgames does the game tree have?

Subgame Perfect Nash Equilibrium

- **An extension of backward induction**
- **A strategy profile is a SPNE if**
  - **It is a NE, and**
  - **For every subgame, the restriction of those strategies to the subgame is also a NE**
- **A solution concept should be consistent with its own application from anywhere in the game where it can be applied**

Finding SPNE

- **Finding SPNE is similar to backward induction**
  - Start from the subgames which start with a node closest to a terminal node
  - Find NE of the subgame
  - Replace the subgame with the NE payoffs and work backwards
  - If there are more than one NE of the subgame, repeat this for each subgame
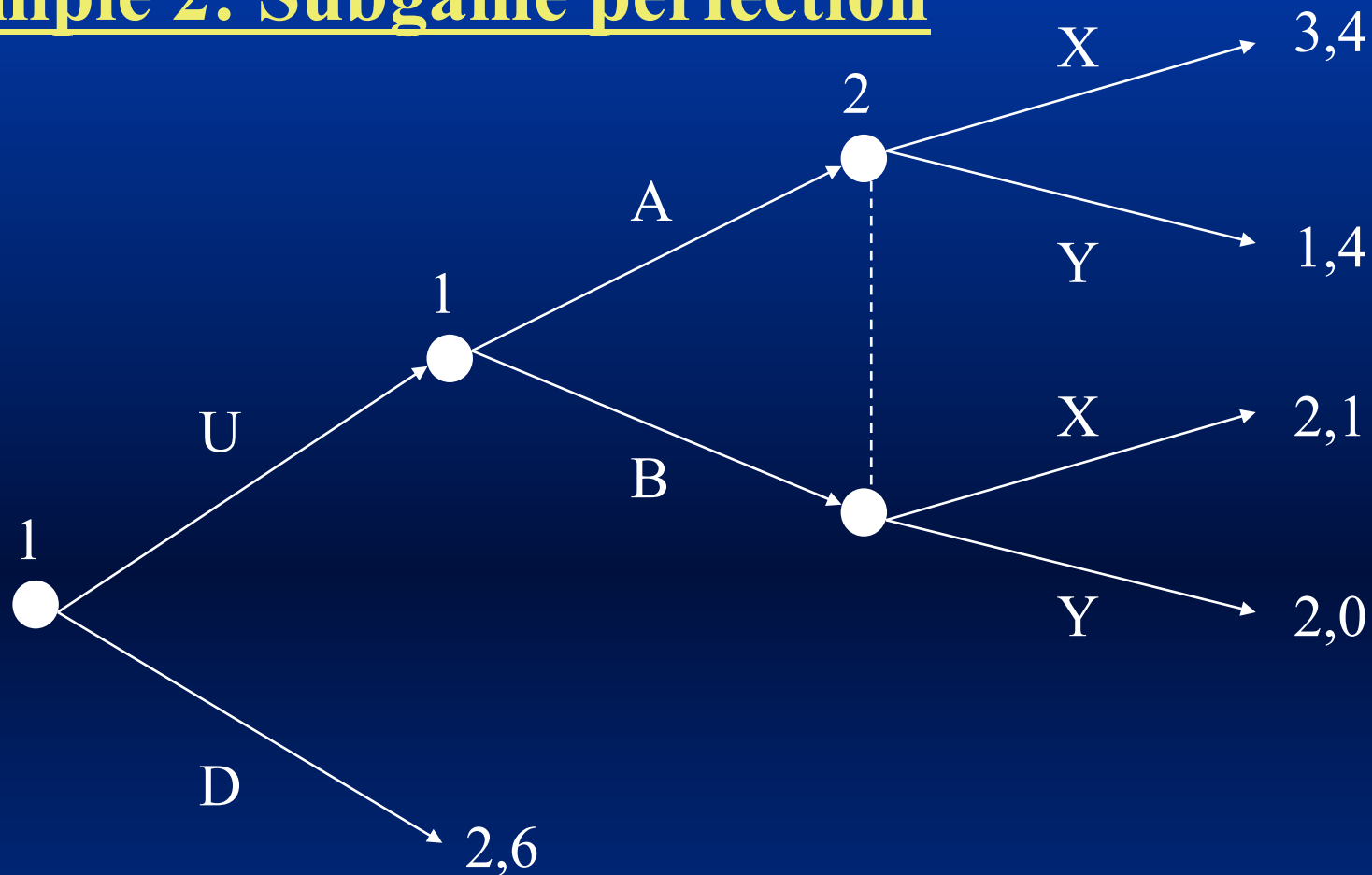
Example 1

- **What are the NE and SPNE of this game?**

Diagrams courtesy: Dr. Tayfun Sönmez
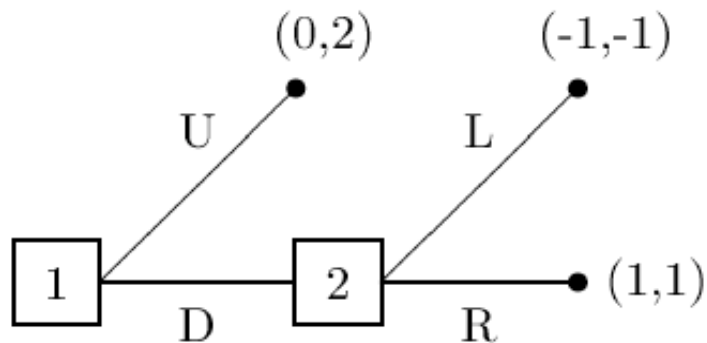
- Two NE: (D, R) and (U, L)
- How many proper subgames?
- Consider the proper subgame: the unique NE of the subgame is R. Therefore, (U; L) is not a SPNE
- Hence (D; R) is the only SPNE
- Note SPNE coincides with the outcome obtained from backward induction

Example 2: Subgame perfection

How many subgame and proper subgames
does this game have?

Example 2: Normal Form of the Entire Game:

	X	Y
UA	**3,4**	**1,4**
UB	**2,1**	**2,0**
DA	**2,6**	**2,6**
DB	**2,6**	**2,6**

2

1

3 pure strategy NE: (UA, X), (DA, Y) and (DB, Y)

Example 2: Normal Form of the Proper Subgame:

	X	Y
A	**3,4**	**1,4**
B	**2,1**	**2,0**

One pure strategy NE: (A, X).
The only SPNE for the game is (UA, X).

Example 3. Price War
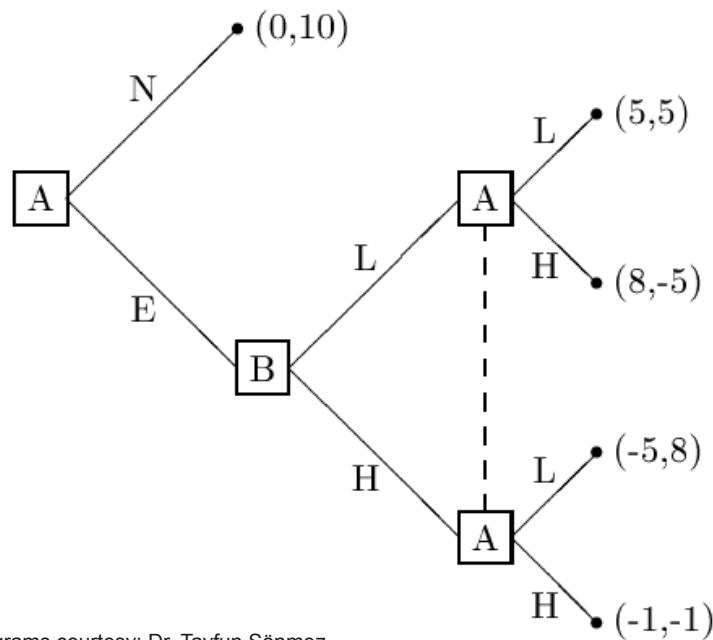
Diagrams courtesy: Dr. Tayfun Sönmez

- **NE: (NH; H) and (NL; H)**
- **How many proper subgames?**
- **Which of the NE are subgame perfect?**
  - Unique NE of subgame is (H; H)
  - Since (L; H) is not a NE of the subgame, (NL; H) is not a SPNE
  - The only SPNE is (NH; H)
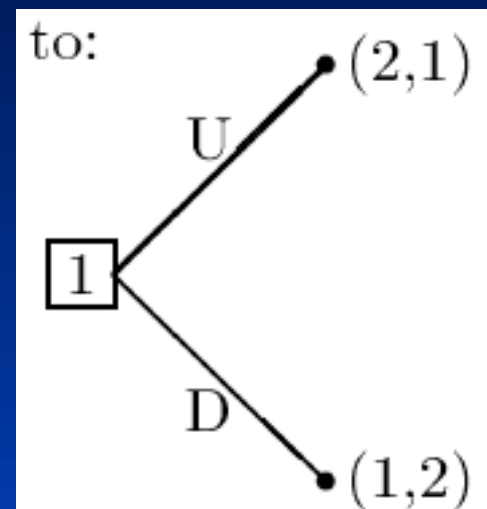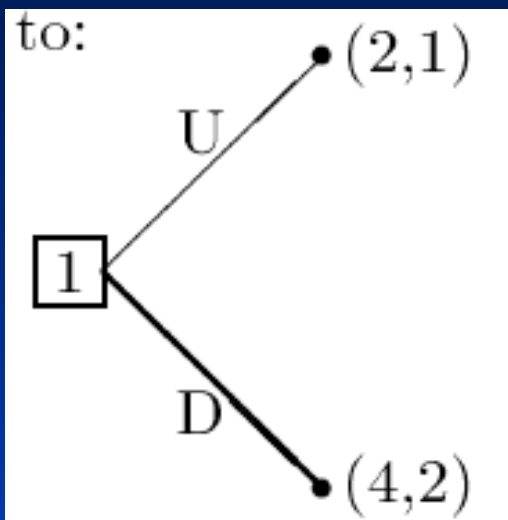
Example 4: Multiple SPNE

Example:
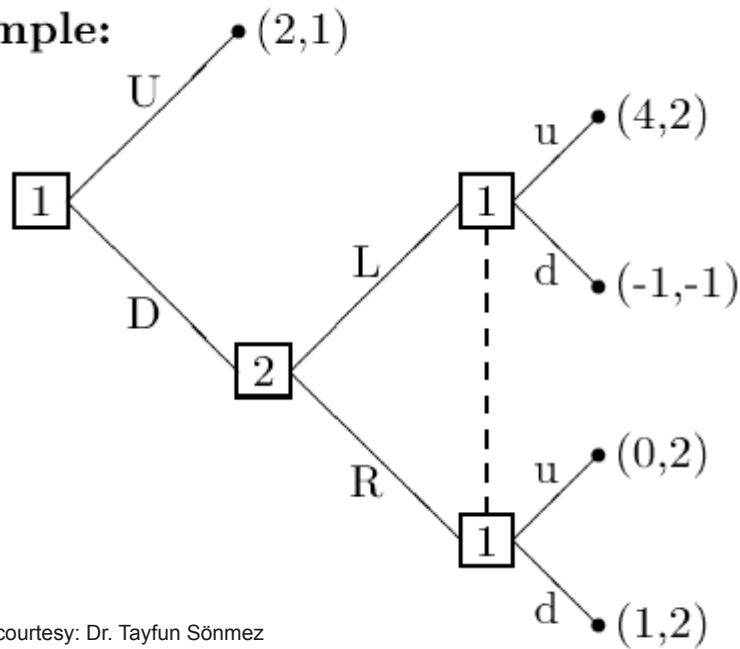
Diagrams courtesy: Dr. Tayfun Sönmez

- **1st subgame**
- **Two NE:**
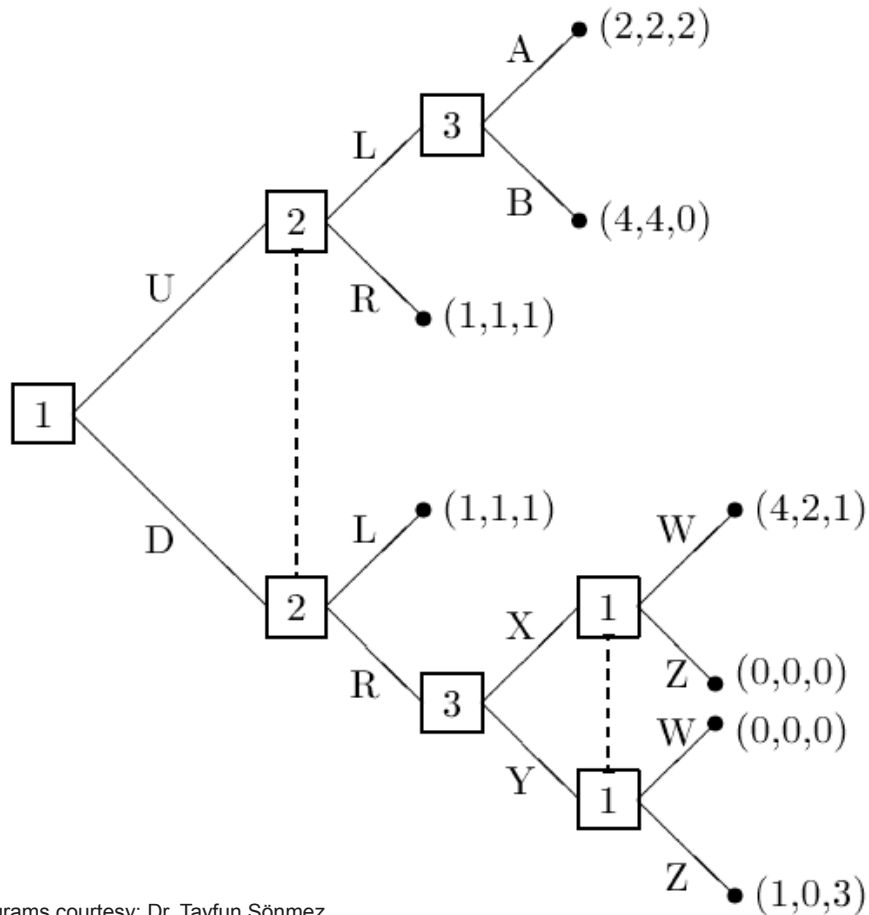 - **(u; L) yielding (4, 2)**
  - **(d; R) yielding (1, 2)**

		Player 2	
		L	R
Player 1	u	4,2	0,2
	d	-1,-1	1,2

Example 4: Multiple SPNE

- **Case 1: replacing subgame with payoff from (u; L)**
- **First SPNE: (Du; L) yielding (4, 2)**

- **Case 2: replacing subgame with payoff from (d, R)**
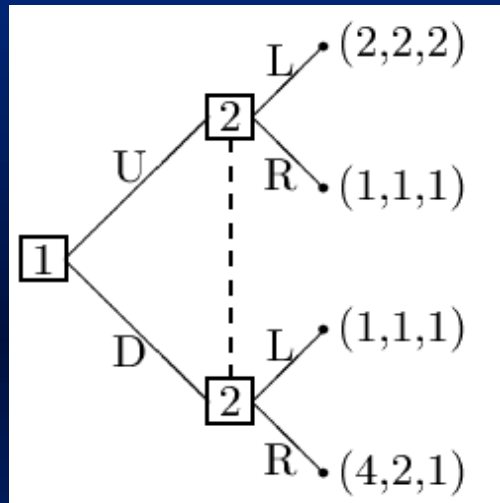- **Second SPNE: (Ud; R) yielding (2, 1)**

Example 5

Diagrams courtesy: Dr. Tayfun Sönmez

- **2 proper subgames**
- **1st subgame: 1-person decision problem. NE yields (2, 2, 2)**
- **2nd subgame yields two NE**

		Player 3	
		X	Y
Player 1	W	4,(2),1	0,(0),0
	Z	0,(0),0	1,(0),3

Example 5: 3 SPNE

- ## Case 1

- ## Case 2:

SPNE: (WU; L; AX) yields (2, 2, 2)
(DW; R; AX) yields (4, 2, 1)
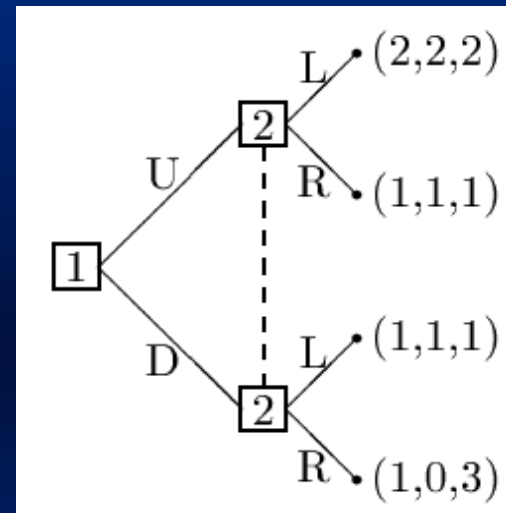
SPNE: (UZ; L; AY) yields (2, 2, 2)

King Solomon's Dilemma

- **Two women claim a baby**
- **Each knows who the true mother is**
- **King Solomon may assign the baby to one, or to neither, and may also impose fines on them, m_1, m_2**
- **Woman i's payoff is**
  - V_h-m_i if she is the true mother
  - V_l-m_i otherwise

    where $V_h > V_l > 0$

King Solomon's Dilemma: an ICD Solution

- **1 is asked if the baby is hers**
  - If 1 says no, baby is given to woman 2 and no fine is imposed
  - If 1 says yes, woman 2 is asked if 2 is the mother
    - » If 2 says no, baby is given to 1 and no fine is imposed
    - » If 2 says yes, 2 gets the baby and
      - 2 pays a fine of M, where $V_h > M > V_l$
      - 1 pays a small fine f > 0

- **What's the SPNE?**

Summary: SPNE

- **A SPNE is a Nash equilibrium**
  - A refinement of NE
- **For games of perfect information, backward induction yields SPNE**
- **The procedure we described is usually the best procedure for *finite* games**

Homework Assignment

- **Chapter 15: #1, 2, 3, 5**