Bargaining

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Some material in this lecture drawn from http://gametheory.net/lectures/level.pl
Bargaining Problems

(Watson Chapter 18)
Bargaining: Value Creation and Division

• Value creation
  – Trade creates value
  – Gains from trade

• Value division
  – Parties jointly decide how to divide the value
  – Bargaining strengths
  – Negotiation procedures
  – Greater contracting environment
• Example: partnership formation
  – Players 1, 2
  – If form partnership, payoff vector (4, 6)
  – If not, payoff vector (2, 2)

• Bargaining set: set of alternatives for a given bargaining problem
  \[ V = \{(4, 6), (2, 2)\} \]

• Default outcome (or disagreement point)
  \[ d = (2, 2) \]
Representation

• Monetary transfer, $t$
• Outcome, $z$
  $z=1$: forming partnership
  $z=0$: no partnership
• Transferable utility
  $u_1 = v_1(z) + t$
  $u_2 = v_2(z) - t$
• Efficient outcomes: max joint value
The bargaining set in the partnership example

- \((4+t, 6-t)\)
- \((2+t, 2-t)\)
- \((4,6)\)
- \((6,4)\)
- \(d_1 = 2\)
- \(d_2 = 2\)
Joint Value and Surplus

• For any z and t, *joint value* is
  \[ [v_1(z) + t] + [v_2(z) - t] = v_1(z) + v_2(z) \]

• *Surplus* of an agreement is defined as the difference between the joint value of the contract and the default:
  \[ v_1(z) + v_2(z) - d_1 - d_2 \]

• *Bargaining power*: bargaining weight

\[ \pi_i: \text{proportion of surplus obtained by player } i \]
Standard Bargaining Solution

• Efficient outcome:

\[ v^* = v_1(z) + v_2(z) \]

maximum payoff

• Players negotiate over the surplus:

\[ v^* - d_1 - d_2 \]

• Standard bargaining solution (Nash)

\[ u_1 = d_1 + \pi_1(v^* - d_1 - d_2) \]
Simple Bargaining Games

Using Noncooperative Game Theory
(Watson Chapter 19)
What determines a player’s bargaining power (weight)?

• Importance of rules:
  The rules of the game determine the outcome

• Diminishing pies:
  The importance of patience

• Estimating payoffs:
  Trust your intuition
Ultimatum Games: Power to the Proposer

• Consider the following bargaining game (over a cake):
  • I name a take-it-or-leave-it split.
  • If you accept, we trade
  • If you reject, no one eats!
  • Under perfect information, there is a simple SPNE
Suppose I can only propose three divisions, (my share, your share):

- \((\frac{1}{4}, \frac{3}{4})\)
- \((\frac{1}{2}, \frac{1}{2})\)
- \((\frac{3}{4}, \frac{1}{4})\)

• Draw the extensive form

• Solve for the SPNE
Ultimatum Bargaining: continuous version

Bargaining set; disagreement point
Ultimatum Bargaining: continuous version

Player j: accept if m > 0;
Player i: offer the smallest possible m.
SPNE: \{m=0; accept all offers\}
Proposer keeps all profits.
Cake Cutting: changing the rules

• Suppose I get to cut the cake in one of three different ways (as before)
• And you get to pick which part is yours
• Draw the extensive form
• Solve for the SPNE
Two-Period Alternating Offer Games: Power to the Patient

• In general, bargaining takes on a “take-it-or-counteroffer” procedure

• If time has value, both parties prefer trade earlier to trade later

• E.g. Labor negotiations –
  Later agreements come at a price of strikes, work stoppages, etc.

• Delays imply less surplus left to be shared among the parties
Two Stage Bargaining

- Bargaining over division of a cake
- I offer a proportion, \( m \), of the cake to you
- If rejected, you may counteroffer (and \( \delta \) of the cake remains, the rest melts)
- Discount factor: \( \delta \)
- Payoffs:
  - In first period: \( 1-m, m \)
  - In second period: \( \delta(1-m), \delta m \)
Bargaining set and disagreement point for 2-stage game
Extensive Form
Since period 2 is the final period, this is just like a take-it-or-leave-it offer: 
  – You will offer me the smallest piece that I will accept, leaving you with all of $\delta$ and leaving me with almost 0

What do I do in the first period?
Backward Induction

• Give you at least as much surplus
• Your surplus if you accept in the first period is 1-m

• Accept if:
  Your surplus in 1st period $\geq$ Your surplus in 2nd period

  $m \geq \delta$
Backward Induction

• If there is a second stage, you get $\delta$ and I get 0.

• You will reject any offer in the first stage that does not offer you at least $\delta$.

• In the first period, I offer you $\delta$.

• Note: the more patient you are (the slower the cake melts) the more you receive now!
First or Second Mover Advantage?

• Are you better off being the first to make an offer, or the second?
Example: Cold Day

- If $\delta = 4/5$ (20% melts)

- Period 2: You offer a division of 1,0
  » You get all of remaining cake = 0.8
  » I get 0 = 0

- In the first period, I offer 80%
  » You get 80% of whole cake = 0.8
  » I get 20% of whole cake = 0.2

Source: Mike Shor, gametheory.net
Example: Hot Day

• If $\delta = 1/5$ (80% melts)

• Period 2: You offer a division of 1,0
  » You get all of remaining cake = 0.2
  » I get 0 = 0

• In the first period, I offer 20%
  » You get 20% of whole cake = 0.2
  » I get 80% of whole cake = 0.8

Source: Mike Shor, gametheory.net
First or Second Mover Advantage?

• When players are impatient (hot day)
  First mover is better off
  – Rejecting my offer is less credible since we both lose a lot

• When players are patient (cold day)
  Second mover better off
  – Low cost to rejecting first offer

• Either way – if both players think through it, deal struck in period 1

Source: Mike Shor, gametheory.net
Don’t Waste Cake

• In any bargaining setting, strike a deal as early as possible!

• Why doesn’t this happen?
  – Reputation building
  – Lack of information

Source: Mike Shor, gametheory.net
Uncertainty in Civil Trials

• Plaintiff sues defendant for $1M
• Legal fees cost each side $100,000
• If each agrees that the chance of the plaintiff winning is ½:
  » Plaintiff: $500K - $100K = $ 400K
  » Defendant: -$500K - $100K = -$600K
• If simply agree on the expected winnings, $500K, each is better off

Source: Mike Shor, gametheory.net
Uncertainty in Civil Trials

• What if both parties are too optimistic?

• Each thinks that his or her side has a $\frac{3}{4}$ chance of winning:
  » Plaintiff: $750K - $100K = $650K
  » Defendant: $250K - $100K = $-350K

• No way to agree on a settlement!

Source: Mike Shor, gametheory.net
Lessons

• Rules of the bargaining game uniquely determine the bargaining outcome

• Which rules are better for you depends on patience, information

• What is the smallest acceptable piece? Trust your intuition

• Delays are always less profitable: Someone must be wrong

Source: Mike Shor, gametheory.net
Homework Assignment

- Chapter 19: #1, 2, 7, 8