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Lecture 5:

User-User Recommender

SI583: Recommender Systems



Generating recommendations

- Core problem: predict how much a person “Joe” (is likely to) like an item “X”
- Then, can decide to recommend most likely successes, filter out items below a threshold, etc.



user	item	A	B	C	...				X
Joe		7	4		4	2		5	?
Sue		7	5	6	5			6	8
John		2		3		7			2



User-User recommenders: Intuition

- Assumption: If Joe and another user agreed on other items, they are more likely to agree on X
- Collaborative filtering approach:
 - For each user, find how similar that user is to Joe on other ratings
 - Find the pool of users “closest” to Joe in taste
 - Use the ratings of those users to come up with a prediction



User-user algorithm: Details to be formalized

- How is similarity measured?
 - how are ratings normalized?
- How is the pool of neighbors selected?
- How are different users' ratings *weighted* in the prediction for Joe?



CF Algorithms in the Literature

- Sometimes classified as *memory-based vs. model-based*
- *Model based*: statistically predict an unknown rating
 - Fit a statistical model, then estimate
 - E.g., SVD
- *Memory-based*: ad-hoc use of previous ratings
 - No explicit class of models, although sometimes retrofit
 - E.g., user-user, item-item



Measures of similarity

user	item	A	B	C	D	...				
Joe		7	3	7	3					
Sue		6	4	6	4					
John		7	7	7	7					
Amy		9	2	3	2					
Bob		7	3							



Some possible similarity metrics

For all our metrics: focus on the ratings on items that both i and j have rated

- *Similarity(i,j) = number of items on which i and j have exactly the same rating*



Some possible similarity metrics

- $Similarity(i,j)$ = number of items on which i and j have the same rating
 - intuitive objection: we would have $similarity(\text{Joe}, \text{John}) > similarity(\text{Joe}, \text{Sue})$



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- $Similarity(i,j)$ = number of items on which i and j have the same rating
 - intuitive objection: we would have $similarity(\text{Joe}, \text{John}) > similarity(\text{Joe}, \text{Sue})$
- $Similarity(i,j) = (i\text{'s rating vector}) \cdot (j\text{'s rating vector})^T$



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 - intuitive objection: we would have $similarity(\text{Joe}, \text{John}) > similarity(\text{Joe}, \text{Sue})$



Some possibilities..

- Normalize for mean rating:

- Let μ_i = i 's average rating

- Let i 's normalized rating vector

$$\mathbf{x}_i = (\text{rating on A} - \mu_i, \text{rating on B} - \mu_i, \dots)$$

- Define $\text{similarity}(i,j) = \mathbf{x}_i \cdot \mathbf{x}_j^T$



Mean-normalized ratings

user	item	A	B	C	D	...			
Joe		2	-2	2	-2				
Sue		1	-1	1	-1				
John		0	0	0	0				
Amy		5	-2	-1	-2				
Bob		2	-2						



Some possibilities..

■ Normalize for mean rating:

– Let μ_i = i 's average rating

– Let i 's normalized rating vector

$$\mathbf{x}_i = (\text{rating on A} - \mu_i, \text{rating on B} - \mu_i, \dots)$$

– Define $\text{similarity}(i,j) = \mathbf{x}_i \cdot \mathbf{x}_j^T$

■ Objection:

$$\text{similarity}(\text{Joe}, \text{Amy}) > \text{similarity}(\text{Joe}, \text{John})$$



Normalizing for mean and standard deviation

- Normalize for mean rating:
 - Let μ_i = i 's average rating
 - Let i 's normalized rating vector

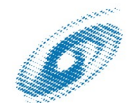
$$\mathbf{x}_i = (\text{rating on A} - \mu_i, \text{rating on B} - \mu_i, \dots)$$
- Then, normalize for standard deviation
 - $\mathbf{z}_i = (1/\sigma)\mathbf{x}_i$
 - where $\sigma = \|\mathbf{x}_i\| = \sqrt{x_i(A)^2 + x_i(B)^2 + \dots / (\# \text{items rated by } i)}$
- Define

$$\text{similarity}(i,j) = \mathbf{z}_i \cdot \mathbf{z}_j^T$$



Mean-std.dev normalized ratings (z-scores)

user	item	A	B	C	D	...				
Joe		1	-1	1	-1					
Sue		1	-1	1	-1					
John		0	0	0	0					
Amy		1.7	-0.7	-0.3	-0.7					
Bob		1	-1							



Normalizing for mean and standard deviation

- Normalize for mean rating:
 - Let μ_i = i's average rating
 - Let i 's normalized rating vector

$$\mathbf{x}_i = (\text{rating on A} - \mu_i, \text{rating on B} - \mu_i, \dots)$$
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 - $\mathbf{z}_i = (1/\sigma)\mathbf{x}_i$
 - where $\sigma = \|\mathbf{x}_i\| = \sqrt{x_i(A)^2 + x_i(B)^2 + \dots / (\#items)}$
- (Modified for different numbers of ratings):

$$\text{similarity}(i,j) = \mathbf{z}_i \cdot \mathbf{z}_j^T / (\#items) \text{ [Pearson correlation coefficient]}$$



Pearson correlation coefficient

- Intuitively: similarity measure that
 - adjusts for different average rating for different users
 - adjusts for different swing magnitudes for different users
 - adjusts for different numbers of common ratings
- Also has a good statistical justification
 - arises naturally in a statistical model..



Correlation: Statistical justification

Statistical model:

- Item w drawn randomly from some space
- Each user's rating is a random variable:
 - i 's rating can be represented by $r_i(w)$
- Goal: Estimate $r_{\text{Joe}}(\text{item})$ from observing $r_{\text{Sue}}(\text{item})$, $r_{\text{John}}(\text{item})$, etc..
- If r_j is independent of r_i , r_j is useless for estimating r_i
- The more correlated r_j is with r_i , the more useful it is (independence \Rightarrow correlation = 0)
- Correlation can be estimated from common ratings



Linear Algebra Representation

- **R**: $[n \times m]$ matrix representing n users' ratings on m items
- **X**: $[n \times m]$ matrix representing ratings normalized by user means
- **Z**: $[n \times m]$ matrix representing z-scores (normalized ratings)



Mathematical representation

- **R**: [$n \times m$] matrix representing n users' ratings on m items
- **X**: [$n \times m$] matrix representing ratings normalized by user means
- **Z**: [$n \times m$] matrix representing z-scores (normalized ratings)

If matrices are complete:

- **C=XX^T** is an [$n \times n$] matrix of covariances
 - C_{ij} / (#items i & j rated) estimates covariance of r_i, r_j
- **P=ZZ^T** is an [$n \times n$] matrix of correlations
 - P_{ij} / (#items i & j rated) estimates correlation of r_i, r_j



Other similarity measures

- Any distance measure between vectors can be used to define a similarity
- e.g., “cosine similarity”
 - treat rating vectors as lines in space, similarity based on how small the angle between i and j is
- How do you decide which one is best?



Other similarity measures

- Any distance measure between vectors can be used to define a similarity
- e.g., “cosine similarity”
 - treat rating vectors as lines in space, similarity based on how small the angle between i and j is
- How do you decide which one is best?
 - intuitively judge what normalizations are important
 - try them out empirically on your data!



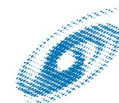
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- How are different users' ratings *weighted* in the prediction for Joe?



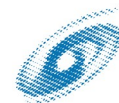
Choosing a pool of neighbors

- **Common approach: *k*-nearest neighbors**
 - Pick up to k users who have rated X , in order of decreasing similarity to X
 - *parameter k is typically about 20-50*
- **Alternative: *Thresholding***
 - Pick all users with correlation coefficients greater than t who have rated X
 - *threshold $t > 0$ is recommended*



Weighting users

- Users' ratings on X are weighted according to computed similarities
- Prediction for Joe is weighted average
 - w_{ij} = Pearson correlation similarity(i,j)
 - predicted $z_{Joe}(x) = \sum_{i \text{ in pool}} w_{i,Joe} z_i(x)$
- Denormalize to compute predicted rating
 - predicted $r_{Joe(x)} = \mu_{Joe} + z_{Joe}(x) \sigma_{Joe}$



Example: Predict Joe's rating for X

user	item	A	B	C	D	...				X
Joe		7	3	7	3					
Sue		6	4	6	4					4
John		7	7	7	7					
Amy		9	2	3	2					6
Bob		7	3							6



Example: z-scores

user	item	A	B	C	D	...				X
Joe		1	-1	1	-1					
Sue		1	-1	1	-1					-1
John		0	0	0	0					
Amy		1.7	-0.7	-0.3	-0.7					0.8
Bob		1	-1							0.6



Example: weights and predictions

- similarity (Amy,Joe) = 0.95
- similarity (Sue,Joe) = 1
- similarity (Bob,Joe) = 1

- *predicted $z_{\text{Joe}}(x) = -0.36$*
- *predicted rating = $5 - 2 * 0.36 = 4.22$*



Recommendations [Herlocker et al, Information and Retrieval, 2002]

Table 8. A tabulation of recommendations based on the results presented in this chapter.

	Recommended	Not recommended
Similarity weighting (Section 5.1)	Pearson correlation	Spearman, entropy, vector similarity, mean-squared difference
Significance weighting (Section 5.2)	Yes	
Selecting neighbors (Section 6)	Set max number of neighbors (potentially in the range of 20–60 nbors)	Weight thresholding
Rating normalization (Section 7.1)	Deviation-from-mean or z-score	No normalization
Weighting neighbor contributions (Section 7.2)	Yes	

