The Finite Element Method for Problems in Physics

Coding Assignment 1

Consider the following differential equation of elastostatics, in strong form:

Find u satisfying

$$(E\, A\, u_{,x})_{,x} + f\, A = 0, \quad \text{in } (0, L),$$

for the following sets of boundary conditions and forcing function ($\bar{f}$ is a constant):

(i) $u(0) = g_1$, $u(L) = g_2$, $f = \bar{f}x$,

(ii) $u(0) = g_1$, $EAu_{,x} = h$ at $x = L$, $f = \bar{f}x$,

where $E = 10^{11}$ Pa, $A = 10^{-4}$ m^2, $\bar{f} = 10^{11}$ Nm^{-4}, $L = 0.1$ m, $g_1 = 0$, $g_2 = 0.001$ m, and $h = 10^6$ N.

Coding Instructions: Write a one-dimensional finite element code to solve the given problem, following these requirements:

- Code (a) linear, (b) quadratic and (c) cubic order Lagrange polynomial basis functions.
- Include a function to calculate the L^2 norm of the error between the finite element solution (u^h) with the exact solution (u), given by $\sqrt{\int_\Omega (u - u^h)^2\, dx}$.
- All integration in K_{local}, F_{local}, and the L^2 norm of the error should be done by Gaussian quadrature (see Lecture 4.11), instead of using the analytical solution to the integrals shown in the lectures.