Consider the following differential equation of elastostatics, in strong form:

Find \( u \) satisfying

\[
(E A u, x)_x + f A = 0, \quad \text{in} \ (0, L),
\]

for the following sets of boundary conditions and forcing function (\( \bar{f} \) is a constant):

(i) \( u(0) = g_1, \ u(L) = g_2, \ f = \bar{f} x, \)

(ii) \( u(0) = g_1, \ EAu, x = h \) at \( x = L, \ f = \bar{f} x, \)

where \( E = 10^{11} \text{ Pa}, \ A = 10^{-4} \text{ m}^2, \ \bar{f} = 10^{11} \text{ Nm}^{-4}, \ L = 0.1 \text{ m}, \ g_1 = 0, \ g_2 = 0.001 \text{ m}, \) and \( h = 10^6 \text{ N}. \)

**Coding Instructions:** Write a one-dimensional finite element code to solve the given problem, following these requirements:

- Code (a) linear, (b) quadratic and (c) cubic order Lagrange polynomial basis functions.
- Include a function to calculate the \( L^2 \) norm of the error between the finite element solution \( (u_h) \) with the exact solution \( (u) \), given by \( \sqrt{\int_\Omega (u - u_h)^2 \, dx}. \)
- All integration in \( K_{local}, F_{local} \), and the \( L^2 \) norm of the error should be done by Gaussian quadrature (see Lecture 4.11), instead of using the analytical solution to the integrals shown in the lectures.