The Finite Element Method for Problems in Physics

Coding Assignment 2

Solve the steady state problem of heat conduction

PDE	$-\nabla \cdot \boldsymbol{j} = \boldsymbol{f}$
Constitutive relation	$\boldsymbol{j} = -\boldsymbol{\kappa}\nabla u$
Neumann b.c.	$-\boldsymbol{j} \cdot \hat{\boldsymbol{n}} = h$ on $\partial\Omega_j$
Dirichlet b.c.	$u = g$ on $\partial\Omega_u$

with the following boundary conditions using the specified meshes and linear basis functions. Use $\bar{\kappa} = 385$ watt.m^{-1}K^{-1}, where $\kappa_{ij} = \bar{\kappa}\delta_{ij}$. Assume $j = 0$ watt.m^{-2} on all edges/surfaces where no temperature/flux conditions are specified.

1. (2D Quadrilateral Mesh): ($x \in [0, 0.03]$, $y \in [0, 0.08]$, use a 15 x 40 element mesh.)
   $u(x) = 300(1 + c_0 x)$ K along $y = 0$ m *(bottom nodeset)* and $u(x) = 310(1 + \hat{c}_0 x^2)$ K along $y = 0.08$ m *(top nodeset)* where $c_0 = \frac{1}{3}K.m^{-1}$, $\hat{c}_0 = 8K.m^{-2}$.

2. (3D Hexahedral Mesh): ($x \in [0, 0.04]$, $y \in [0, 0.08]$, $z \in [0, 0.02]$, use a 8 x 16 x 4 element mesh)
   $u(y, z) = 300(1 + c_0(y + z))$ K along $x = 0$ m *(left nodeset)* and $u(y, z) = 310(1 + c_0(y + z))$ K along $x = 0.04$ m *(right nodeset)*

   where $c_0 = \frac{1}{3}K.m^{-1}$, $\hat{c}_0 = 8K.m^{-2}$.