The Finite Element Method for Problems in Physics

Coding Assignment 2

Solve the steady state problem of heat conduction

\[ \nabla \cdot j = f \]

Constitutive relation

\[ j = -\kappa \nabla u \]

Neumann b.c.

\[ -j \cdot \hat{n} = h \text{ on } \partial \Omega_j \]

Dirichlet b.c.

\[ u = g \text{ on } \partial \Omega_u \]

with the following boundary conditions using the specified meshes and linear basis functions. Use \( \tilde{\kappa} = 385 \text{ watt.m}^{-1}\text{K}^{-1} \), where \( \kappa_{ij} = \kappa \delta_{ij} \). Assume \( j = 0 \text{ watt.m}^{-2} \) on all edges/surfaces where no temperature/flux conditions are specified.

1. (2D Quadrilateral Mesh): \((x \in [0, 0.03], y \in [0, 0.08], \text{ use a } 15 \times 40 \text{ element mesh.})\)
   \[ u(x) = 300(1 + c_0 x) \text{ K along } y = 0 \text{ m (bottom nodeset) and } u(x) = 310(1 + \hat{c}_0 x^2) \text{ K along } y = 0.08 \text{ m (top nodeset)} \text{ where } c_0 = \frac{1}{3} K.m^{-1}, \hat{c}_0 = 8 K.m^{-2}. \]

2. (3D Hexahedral Mesh): \((x \in [0, 0.04], y \in [0, 0.08], z \in [0, 0.02], \text{ use a } 8 \times 16 \times 4 \text{ element mesh})\)
   \[ u(y, z) = 300(1 + c_0 (y+z)) \text{ K along } x = 0 \text{ m (left nodeset) and } u(y, z) = 310(1 + c_0 (y+z)) \text{ K along } x = 0.04 \text{ m (right nodeset)} \]
   \[ \text{ where } c_0 = \frac{1}{3} K.m^{-1}, \hat{c}_0 = 8 K.m^{-2}. \]