The Finite Element Method for Problems in Physics

Coding Assignment 3

Consider the 3D elastostatics problem. Find u such that

PDE	$\sigma_{ij,j} + f_i = 0$ in Ω
Constitutive relation	$\sigma_{ij} = \mathbb{C}_{ijkl}\epsilon_{kl}$
Kinematic relation	$\epsilon_{kl} = \frac{1}{2}\left(\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k}\right)$
Neumann b.c.	$\sigma_{ij}n_j = h_i$ on $\partial\Omega_{h_i}$
Dirichlet b.c.	$u_i = u_i^g$ on $\partial\Omega_{u_i}$

Consider a three-dimensional domain defined by $x_1 = [0,1]$ m; $x_2 = [0,1]$ m; $x_3 = [0,1]$ m (i.e. the unit cube). Use $E = 2.0e11$ Pa and $\nu = 0.3$. Assume traction $h_i = 0$ N.m^{-2} on all surfaces where no other conditions are specified. Use linear basis functions and a 10 x 10 x 10 element mesh for submission.

Apply the following boundary conditions:

$h_1 = h_2 = 0, h_3 = 1.0e9 * x_1$ Pa on the face $x_3 = 1$ m and $u_1 = u_2 = u_3 = 0$ m on the face $x_3 = 0$ m.