Physics 140 – Fall 2007
lecture #14: 23 October

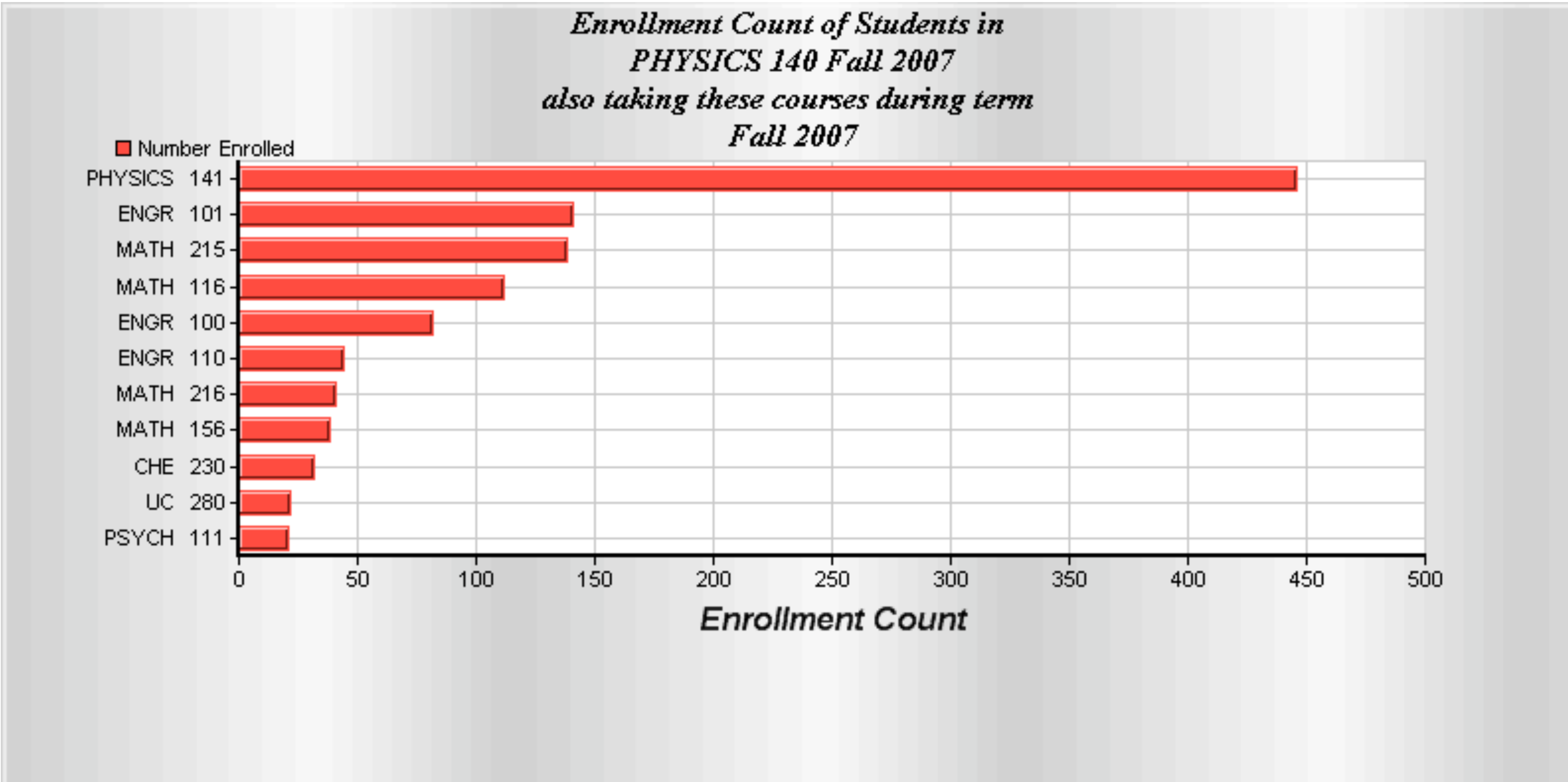
Ch 8 topics:
- center of mass
- rockets

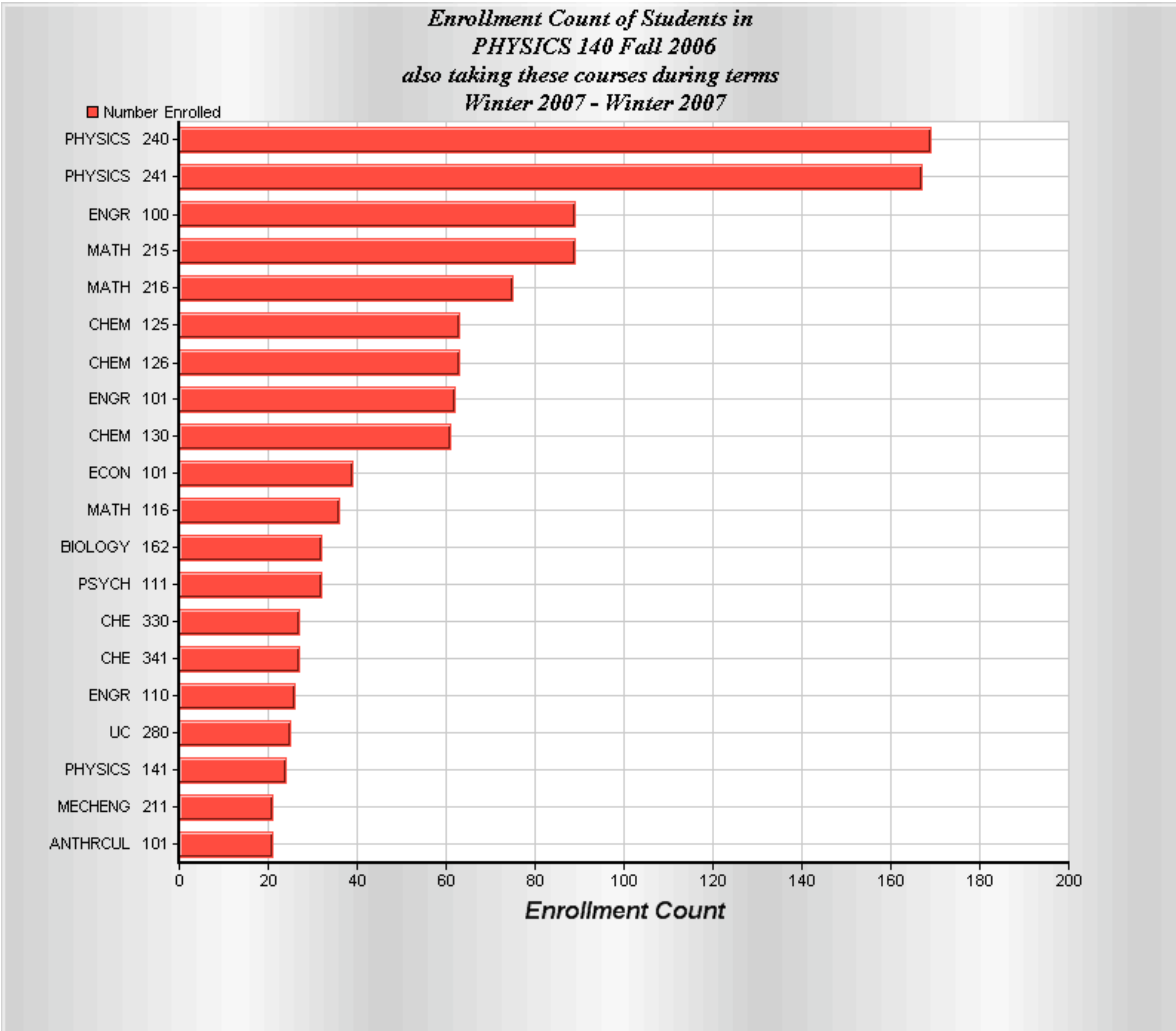
Announcements:
- exam #2 is next Thursday, 1 November, 6:00-7:30pm
- covers Chapters 6-8
- practice exam on course web page under Resources link
- review next Monday evening, 29 October, 8:00-9:30pm

One should not pursue goals that are easily achieved. One must develop an instinct for what one can just barely achieve through one’s greatest efforts.

Source: The Scientific Monthly (1921)

The New Quotable Einstein, A. Calaprice, ed., Princeton Univ. Press, 2005, p. 220.


Enrollment Count of Students in
PHYSICS 140 Fall 2007
also taking these courses during term
Fall 2007
Number Enrolled
PHYSICS 141
ENGR 101
MATH 215
MATH 116
ENGR 100
ENGR 110
MATH 216
MATH 156
CHE 230
UC 280
PSYCH 111
0
50
100
150
200
250
300
350
400
450
500
Enrollment Count



Enrollment Count of Students in
PHYSICS 140 Fall 2006
also taking these courses during terms
Winter 2007 - Winter 2007
Number Enrolled
PHYSICS 240
PHYSICS 241
ENGR 100
MATH 215
MATH 216
CHEM 125
CHEM 126
ENGR 101
CHEM 130
ECON 101
MATH 116
BIOLOGY 162
PSYCH 111
CHE 330
CHE 341
ENGR 110
UC 280
PHYSICS 141
MECHENG 211
ANTHRCUL 101
0
20
40
60
80
100
120
140
160
180
200
Enrollment Count

Center of mass: position, velocity and acceleration

For a pair of objects, one of mass m_1 located at $\boldsymbol{r}_1$ and the other of mass m_2 located at $\boldsymbol{r}_2$, we define the *center of mass position* by a mass-weighted, normalized sum

$$\boldsymbol{r}_{\text{com}} = (m_1 \boldsymbol{r}_1 + m_2 \boldsymbol{r}_2) \ / (m_1 + m_2) \ .$$

If the objects have velocities $\boldsymbol{v}_{\text{i}}$ and accelerations $\boldsymbol{a}_{\text{i}}$ (i=1,2), we similarly define the *center of mass velocity* as

$$\boldsymbol{v}_{\text{com}} = (m_1 \boldsymbol{v}_1 + m_2 \boldsymbol{v}_2) \ / (m_1 + m_2)$$

and *center of mass acceleration* as

$$\boldsymbol{a}_{\text{com}} = (m_1 \boldsymbol{a}_1 + m_2 \boldsymbol{a}_2) \ / (m_1 + m_2) \ .$$

Generalize to n masses by extending the sums

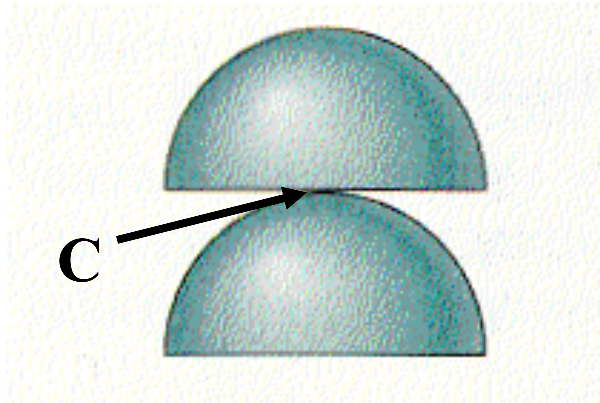
$$M = \sum_{i=1}^{n} m_i$$

$$\vec{r}_{com} = \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{\sum_{i=1}^{n} m_i} = \frac{1}{M} \sum_{i=1}^{n} m_i \vec{r}_i$$

$$\vec{v}_{com} = \frac{1}{M} \sum_{i=1}^{n} m_i \vec{v}_i$$

$$\vec{a}_{com} = \frac{1}{M} \sum_{i=1}^{n} m_i \vec{a}_i$$

In the case of a continuous distribution of mass, the sums are replaced by integrals over the area or volume of the object.



From the symmetry of a uniform sphere, it makes sense that its center of mass is located at its geometric center. But what if we chop a uniform sphere in two and stack the halves as shown above? Where is the vertical position of the center of mass in this arrangement?

1. at the contact point C
2. above the contact point C
3. below the contact point C

COM momentum *is* total system momentum

The total linear momentum of a system of two particles is the same as the system's center of mass linear momentum

$$\boldsymbol{P} = \boldsymbol{p}_1 + \boldsymbol{p}_2 = (m_1 + m_2)\boldsymbol{v}_{\mathrm{com}} = M\,\boldsymbol{v}_{com}$$

For a system of N particles, the total momentum can similarly be expressed as the product of the total mass *M* times the center of mass velocity $\boldsymbol{v}_{com}$

$$\vec{P} = \sum_{i=1}^{n} m_i \vec{v}_i = \sum_{i=1}^{n} \vec{p}_i = M\vec{v}_{\mathrm{com}}$$

Why are these definitions useful?

1) Newton's 2nd law states that the net force changes an object's momentum

$$\vec{F}_{\text{net}} = d\vec{P}/dt$$

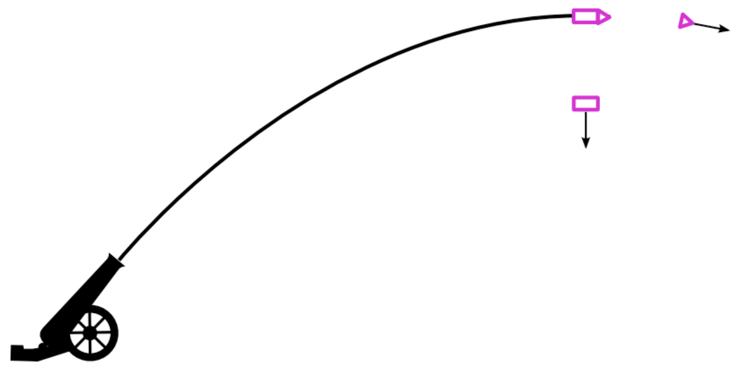
When an object's mass M is constant in time, this is the familiar

$$\vec{F}_{\text{net}} = M\vec{a}_{\text{com}}$$

2) **COM location = "shrink to a dot".** The *center-of-mass position* of an extended object *moves as if it were a point particle* acted on by the object's net force. (Recall the dumbbell demo in lecture.)

3) If a system has *no net momentum* and *no net forces acting on it*, then its center of mass location does not change, *even if* its internal configuration *does* change.

$$(\vec{r}_{\text{com}})_{\text{final}} = (\vec{r}_{\text{com}})_{\text{initial}}$$



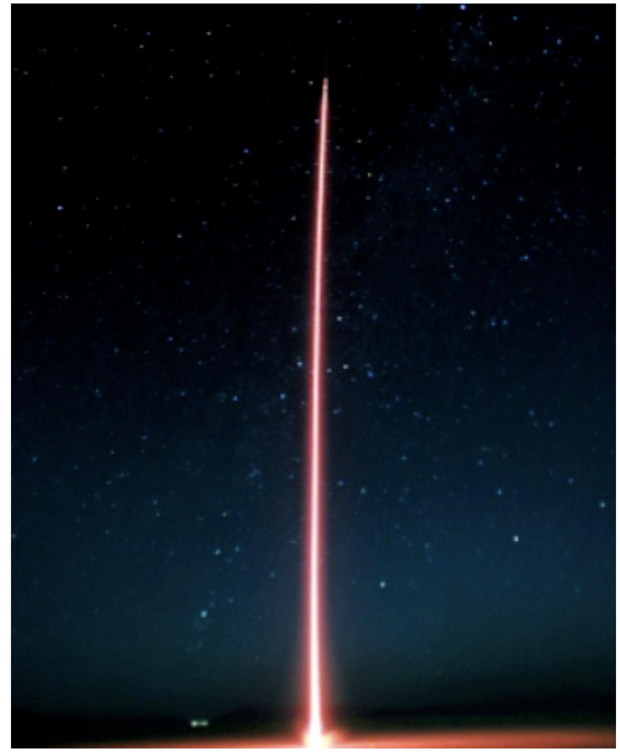
A shell is fired with initial speed v_0 at an angle of 60 degrees above a level, horizontal landscape. At the maximum height of the trajectory, the shell explodes into two pieces of *equal mass*. One piece has zero velocity after the explosion and falls straight down. Where will the second piece land?

1. twice as far as the first piece.
2. three times as far.
3. four times as far.
4. need more information.

Behavior of Rockets

A rocket accelerates by expelling exhaust from the rear of the craft. The recoil from the momentum lost backward to the exhaust propels the spacecraft forward. If the rocket expels gas with velocity v_{ex} at a rate $|dm/dt|$, then the acceleration of the rocket, of time-varying mass m(t), obeys

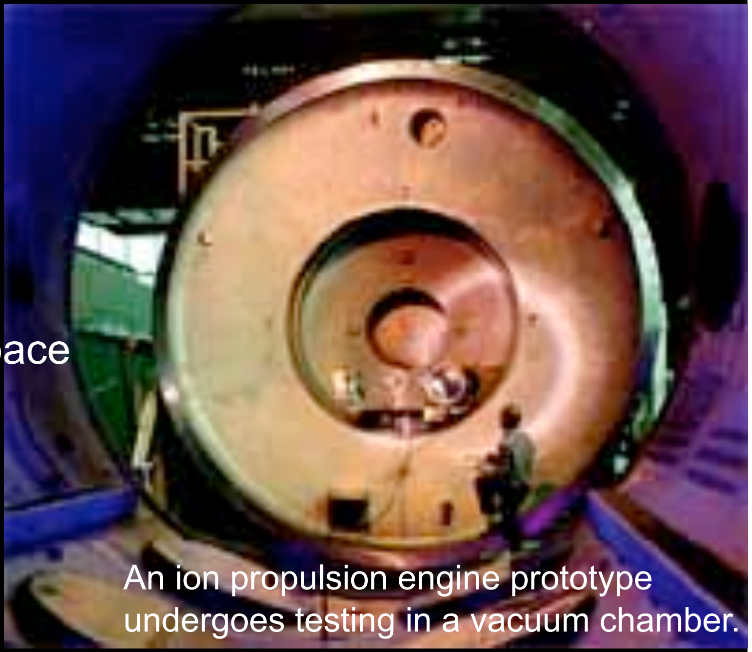
$$m(t)a = \left|\frac{dm}{dt}\right| v_{ex}$$



The rocket loses mass over time $$m(t) = m_0 - \left|\frac{dm}{dt}\right| t$$

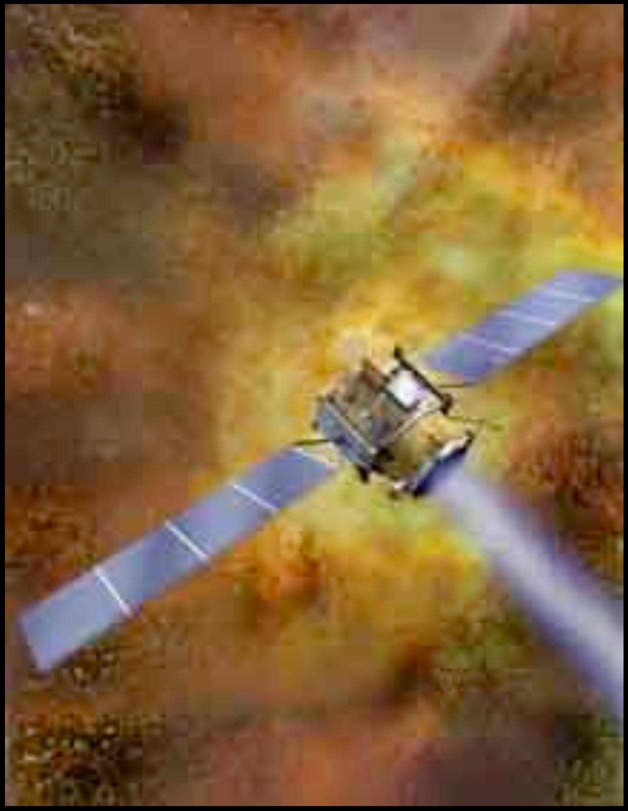
but it speeds up in the process $$v(t) - v_0 = v_{\text{ex}} \ln\left(\frac{m_0}{m(t)}\right)$$

NASA's DEEP SPACE 1 TECHNOLOGY TESTBED MISSION (1998–2001)


An ion propulsion engine prototype undergoes testing in a vacuum chamber.

Source: NASA

Among the exciting revolutionary technologies that Deep Space 1 tested in space were:
SOLAR ELECTRIC ION PROPULSION
(continuous small "reverse collisions" with xenon ions)



Source: NASA

Unlike chemical rocket engines, ion engines accelerate nearly continuously, giving each ion a tremendous burst of speed. The DS1 engine provides about 10 times the specific impulse (ratio of thrust to propellant used) of chemical propulsion.

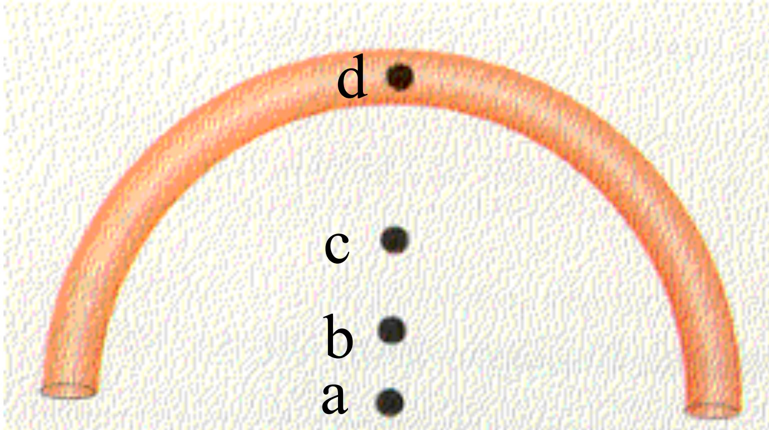
http://nmp.jpl.nasa.gov/ds1/

From a stationary start, is it possible for a rocket to accelerate up to a speed equal to the exhaust speed of its ejected fuel?

1. sure, why not?
2. no way
3. maybe?

Consider a rocket leaving an abandoned space station in deep space. What fraction of a rocket's initial total mass must be consumed in fuel for it to accelerate from a standing start to a speed equal to its exhaust gas speed?

1. 1/e
2. 1-1/e
3. 1/2
4. .999



Which point is most likely to be the center of mass of the uniform semi-circular hoop shown?

1. a
2. b
3. c
4. d

an out of body experience!...

You are in a boat resting on a lake on a perfectly calm day. You fall asleep and lose your oars overboard. Spying two large but unequal size rocks lying in your boat (for ballast? for hitting carp?), you get the brilliant idea of throwing the rocks from the boat away from shore so that your boat will recoil and drift into port. If you throw the rocks with some fixed velocity v_{rel} (relative to you), what scheme will end up giving you and your boat the fastest trip back to shore?

1. throw both rocks simultaneously.
2. throw the lighter, then the heavier.
3. throw the heavier, then the lighter.
4. it doesn't matter how you throw them.