Physics 140 – Fall 2007
lecture #15 : 25 Oct

Ch 9 topics:
• rotational kinematics
• rotational kinetic energy
• moment of inertia

• exam #2 is next Thursday, 1 November, 6:00-7:30pm
• covers Chapters 6-8
• practice exam on CTools site -> Exams & Grading
  bring **two** 3x5 notecards, calculator, #2 pencils

• review next Monday evening, 29 October, 8:00-9:30pm
Center of Mass of an extended, non-uniform object

An object with surface mass density $\sigma(r)$, shown here in 2D, has a total mass given by the integral

$$M = \iint_{\text{object}} dx \, dy \, \sigma(r)$$

Its center of mass is defined by integrals of the mass-weighted positions:

$$x_{\text{com}} = \frac{1}{M} \iint_{\text{object}} dx \, dy \, \sigma(r)x$$

$$y_{\text{com}} = \frac{1}{M} \iint_{\text{object}} dx \, dy \, \sigma(r)y$$
Rotational kinematics

To describe rotational motion, we begin with the \textit{angular position} $\theta$ (in radians) measured relative to an (arbitrary) reference angle.

really $2n\pi$, $n=0, \pm 1, \pm 2, \pm 3, \ldots$
Rotational kinematics

A change in angular position, $\Delta \theta$, during a time interval $\Delta t$ implies a non-zero average angular velocity

$$\omega_{\text{avg}} = \frac{\Delta \theta}{\Delta t}$$

A change in angular velocity, $\Delta \omega$, defines an average angular acceleration

$$\alpha_{\text{avg}} = \frac{\Delta \omega}{\Delta t}$$

The limit $\Delta t \to 0$ defines instantaneous measures for these

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt}$$

The snapshots of the ball are shown at fixed time intervals. Note how the angular displacement between images grows in time, implying an angular acceleration $\alpha > 0$. 

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The magnitudes of translational quantities - displacement, velocity and tangential acceleration \(( l, v, a_{\text{tan}} )\) - are equal to the angular equivalent measures \(( \theta, \omega, \alpha \) multiplied by the distance \( r \) from the rotation axis.

At fixed \( r \), the components of:
- displacement
- tangential velocity
- tangential accel. \( a_{\text{tan}} \)

**TABLE 8.2 Linear and angular kinematic parameters**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l = r\theta )</td>
<td>( l \text{ (m)} ) ( \theta \text{ (rad)} )</td>
</tr>
<tr>
<td>( v = r\omega )</td>
<td>( v \text{ (m/s)} ) ( \omega \text{ (rad/s)} )</td>
</tr>
<tr>
<td>( a_T = r\alpha )</td>
<td>( a_T \text{ (m/s}^2) ) ( \alpha \text{ (rad/s}^2) )</td>
</tr>
</tbody>
</table>
The kinematic equations developed in Chapter 2 for translational motion also apply to rotational motion. (see Table 9.1 in Y&F)

<table>
<thead>
<tr>
<th>Rotational Motion [ (\alpha = \text{constant}) ]</th>
<th>Linear Motion [ (a = \text{constant}) ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \omega = \omega_0 + \alpha t ]</td>
<td>[ v = v_0 + at ]</td>
</tr>
<tr>
<td>[ \theta = \frac{1}{2} (\omega_0 + \omega) t ]</td>
<td>[ x = \frac{1}{2} (v_0 + v) t ]</td>
</tr>
<tr>
<td>[ \theta = \omega_0 t + \frac{1}{2} \alpha t^2 ]</td>
<td>[ x = v_0 t + \frac{1}{2} at^2 ]</td>
</tr>
<tr>
<td>[ \omega^2 = \omega_0^2 + 2 \alpha \theta ]</td>
<td>[ v^2 = v_0^2 + 2ax ]</td>
</tr>
</tbody>
</table>
Two ladybugs rest without slipping on a rotating platter that is increasing its angular velocity. Ladybug A is closer to the rotation axis than bug B.

Which statement correctly describes the relationship between the bugs’ angular accelerations ($\alpha$) and centripetal accelerations ($a_{\text{rad}}$)?

1) $\alpha_A > \alpha_B$ and $a_{\text{rad},A} > a_{\text{rad},B}$
2) $\alpha_A < \alpha_B$ and $a_{\text{rad},A} < a_{\text{rad},B}$
3) $\alpha_A = \alpha_B$ and $a_{\text{rad},A} < a_{\text{rad},B}$
4) $\alpha_A = \alpha_B$ and $a_{\text{rad},A} = a_{\text{rad},B}$
5) $\alpha_A = \alpha_B$ and $a_{\text{rad},A} > a_{\text{rad},B}$
Negotiating circular motion at tangential speed \( v_t \) around a circular arc of radius \( r \) still requires a **radial component** of acceleration

\[
a_{\text{rad}} = \frac{v_t^2}{r} = \omega^2 r
\]
directed towards the center of the circle. This component changes the direction of the velocity, keeping it tangent to the circle.

The **tangential component** of acceleration

\[
a_{\text{tan}} = \alpha r
\]
changes the speed, \( \frac{dv_t}{dt} = a_{\text{tan}} \).
Rotational kinetic energy and moment of inertia

A set of masses $m_i$ uniformly rotating with angular velocity $\omega$ about some fixed axis $A$ possesses a kinetic energy defined by

$$K = \frac{1}{2} \sum_i m_i v_i^2 = \frac{1}{2} \sum_i m_i r_i^2 \omega^2$$

where $r_i$ is the distance from the $i^{th}$ mass to the rotation axis.

For such a set of mass, or for a continuous body, we define the moment of inertia $I$ about the specified axis $A$ as

$$I = \sum_i m_i r_i^2$$

Then the rotational kinetic energy can be written as

$$K = \frac{1}{2} I \omega^2$$
A given object has only one mass \( m \), but **many** moments of inertia \( I \), depending on the location and orientation of the rotation axis.

Note: this graphic assumes an object of unit mass \( (M=1) \).

Refer to Table 9.2 in YF for a similar list.

Source: Undetermined
The three spheres above have the same mass $M$ and the same radius $R$. Sphere $B$ is hollow, $A$ and $C$ are solid. Sphere $C$ rotates about an axis adjacent to its edge while spheres $A$ and $B$ rotate about their centers. All rotate at the same angular velocity. Rank the spheres according to their rotational kinetic energy, largest to smallest.

1. A, B, C
2. B, A, C
3. A, C, B
4. C, B, A

Source: Undetermined