# Live from Ann Arbor,
# it's Tuesday Morning!

Ch 13 topics:

- restoring forces produce oscillations
- simple harmonic motion (SHM)
- damped harmonic motion
- natural frequency, driven oscillations and resonance

Midterm exam #3 is next Thursday, 29 November
covers chapters 9-12 (rotation through gravity)
bring **three** 3x5 notecards, calculator, #2 pencils

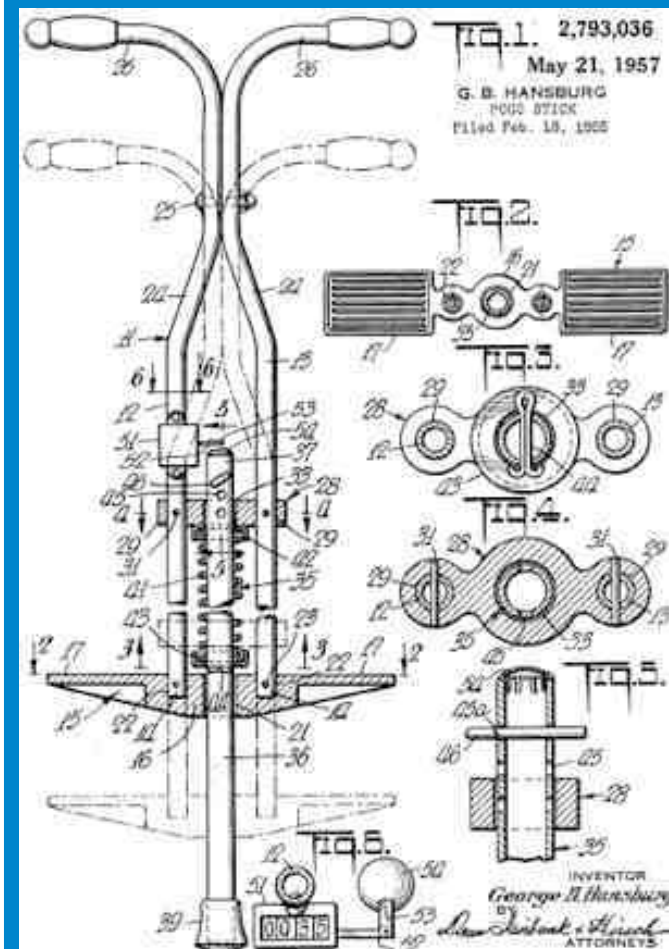
- Review on Monday, 26 Nov, 8:00-9:30pm

# Which is your favorite FM station?

A: 88.1
B: 88.3
C: 88.7
D: 95.5
E: 97.9
F: 107.1

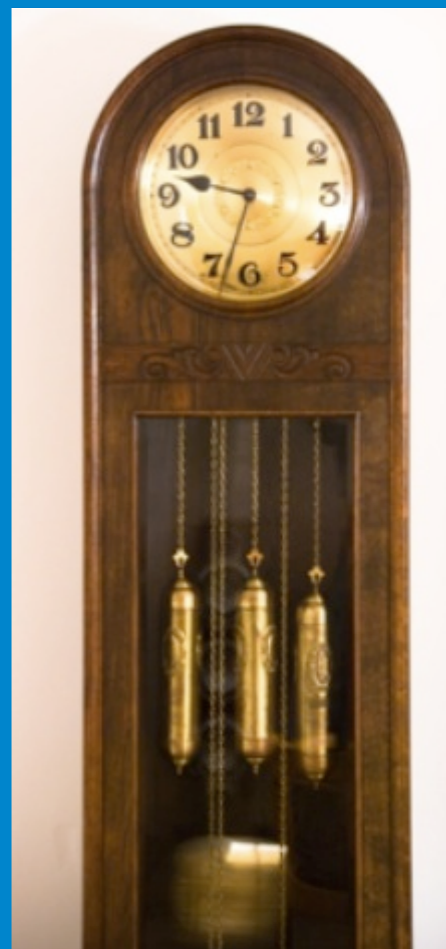
# Things that Oscillate

mass on spring



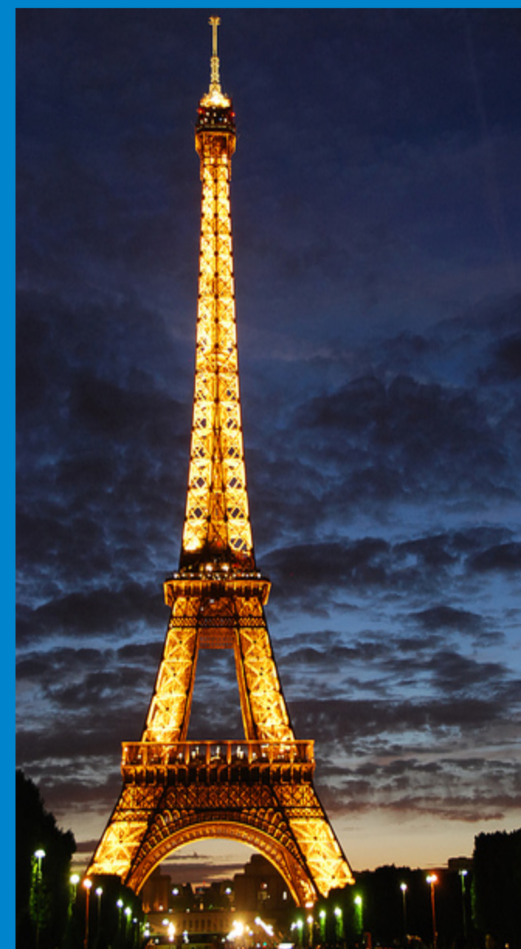



pendulum



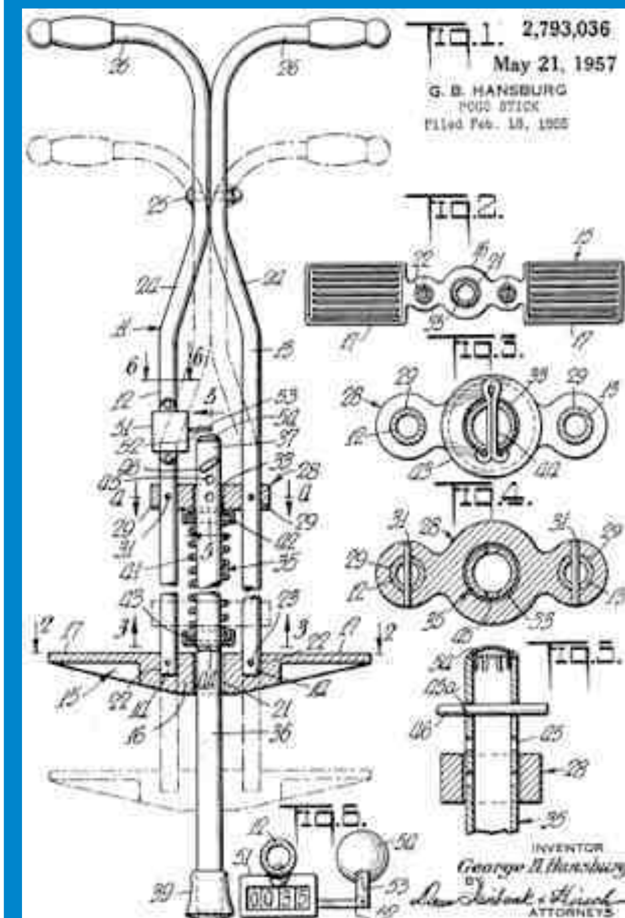


$e^-$ in antenna

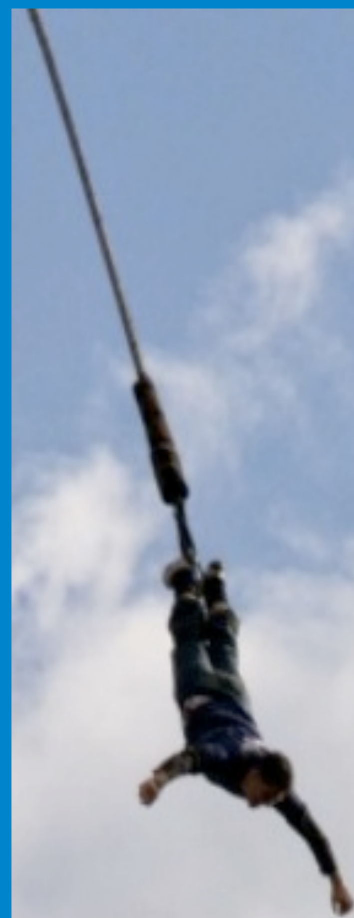


# Things that Oscillate: I

masses on springs




Source: US Patent





Source: Undetermined

A ***restoring force*** leads to oscillations about a point of equilibrium.

http://en.wikipedia.org/wiki/File:Muelle.gif

# Linear Restoring Forces and Simple Harmonic Motion

A ***linear restoring force*** tends to push a system back toward a point of stable equilibrium, with a magnitude that varies linearly with the displacement away from equilibrium. An example is Hookes' law for an ideal spring

$$F = -kx$$

Applying Newton's second law gives a second-order ordinary differential equation

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

the solution of which is a ***sinusoidal variation of position*** in time

$$x(t) = x_m \cos(\omega t + \phi) \qquad (\omega = \sqrt{k/m})$$

Any system with displacement following this form is said to be undergoing ***simple harmonic motion* (SHM)**.

# Conditions for SHM

*Any* system for which the acceleration varies with the negative of the displacement will exhibit SHM. The coefficient between $a$ and $x$ defines the square of the angular frequency $\omega^2$.

$$a(x) = -\omega^2 x \quad \Longleftrightarrow \quad x(t) = x_m \cos(\omega t + \phi)$$

## Descriptive features of SHM

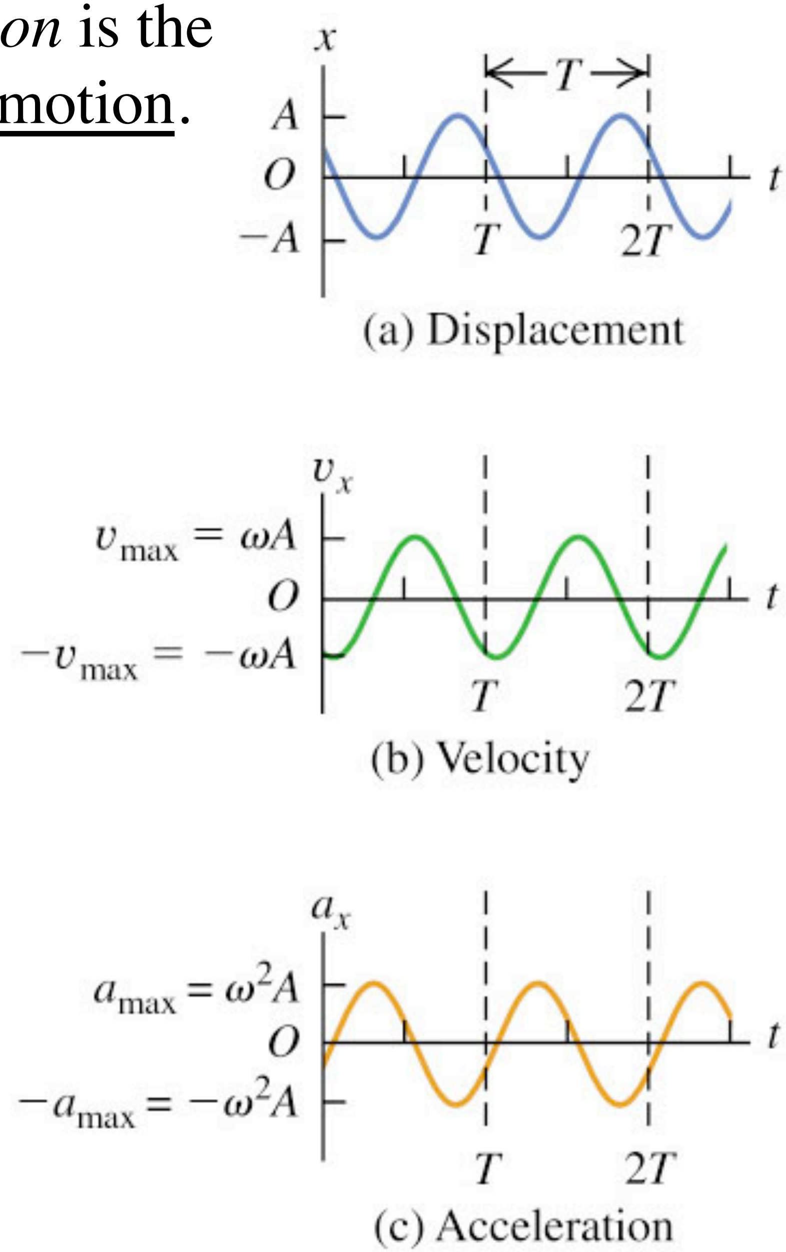
Although the causes of SHM will vary from one system to another, the sinusoidal variation is a common element. All solutions are directly characterized by three features:

- $x_m$ : **maximum displacement amplitude** (or *amplitude*)
- $\omega$ : **angular frequency**
- $\phi$ : **phase constant** (or *phase angle*)

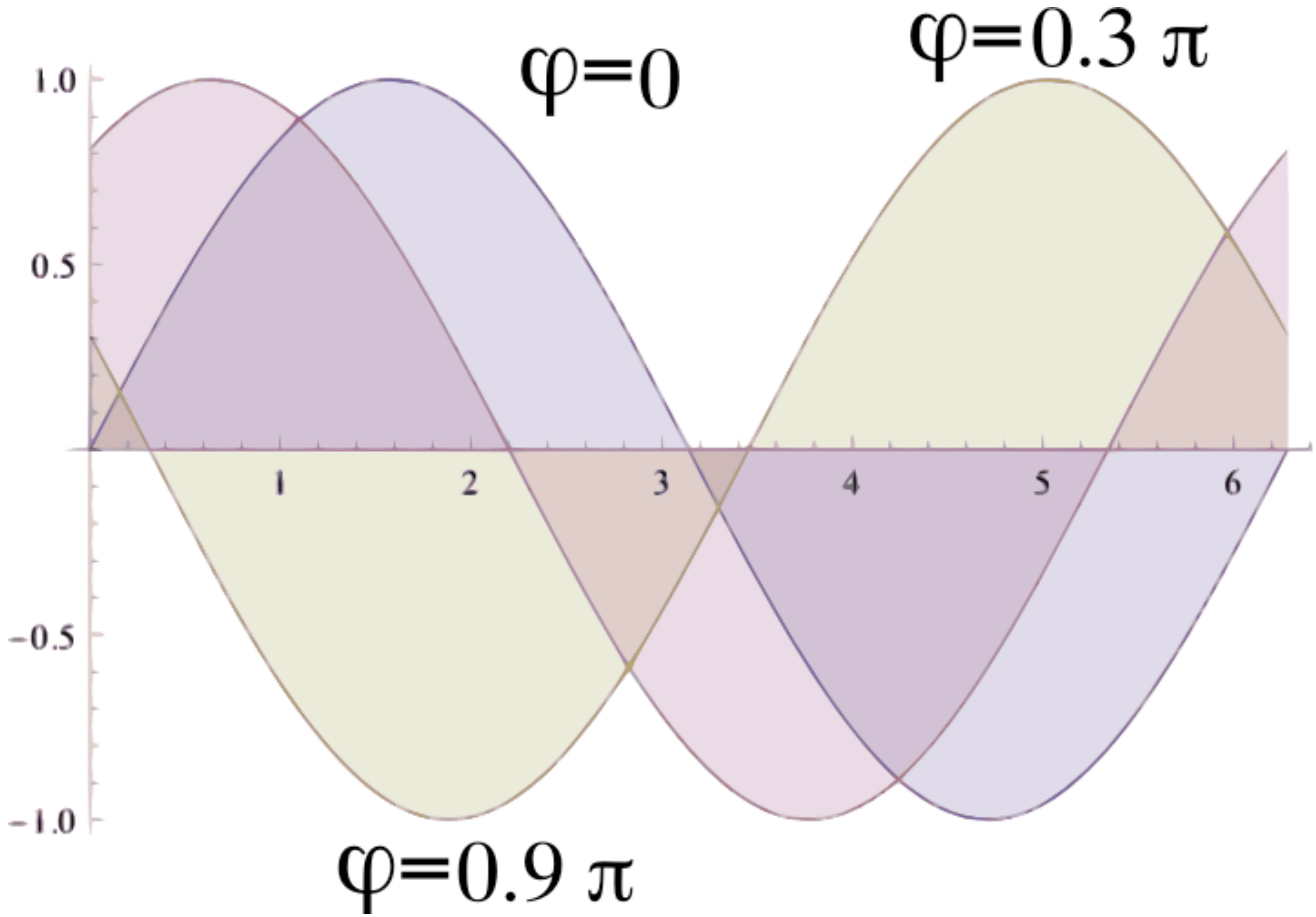
and $\omega$ can alternately be specified by either of the following:

- $f$ : **frequency,** $f = \omega/2\pi$ **measured in Hertz (1 Hz = 1s$^{-1}$)**
- $T$ : **period,** $T = 1/f = 2\pi/\omega$

The behavior of *simple harmonic motion* is the same as a linear projection of circular motion.



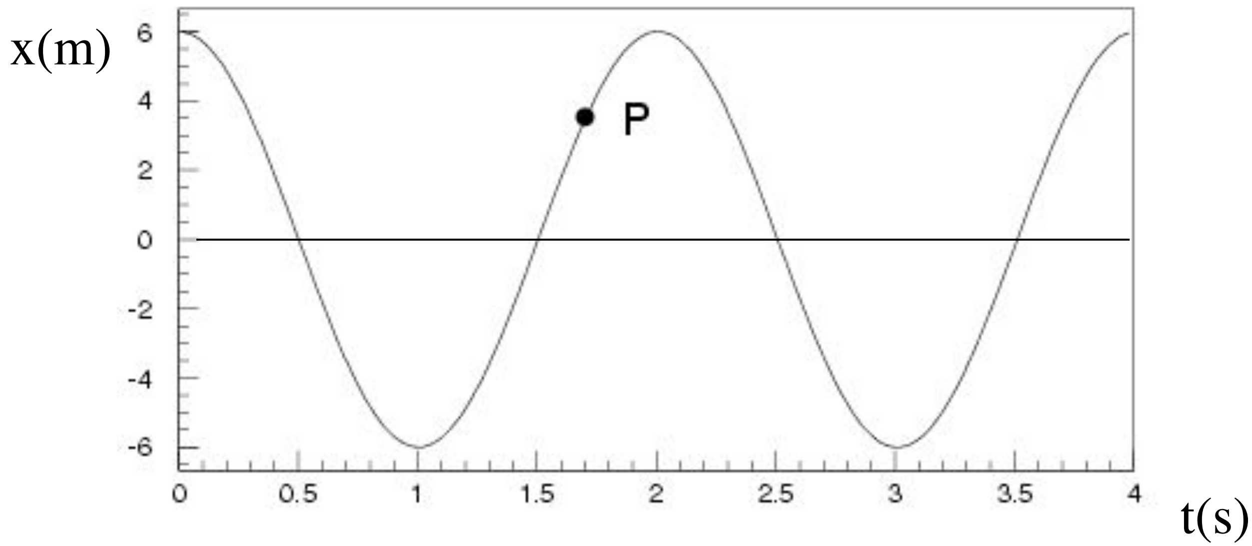

http://en.wikipedia.org/wiki/File:Simple_Harmonic_Motion_Orbit.gif




$$x(t) = x_m \cos(\omega t + \phi)$$

The behavior at $t = 0$ defines the ***phase constant*** $\boldsymbol{\phi}$.

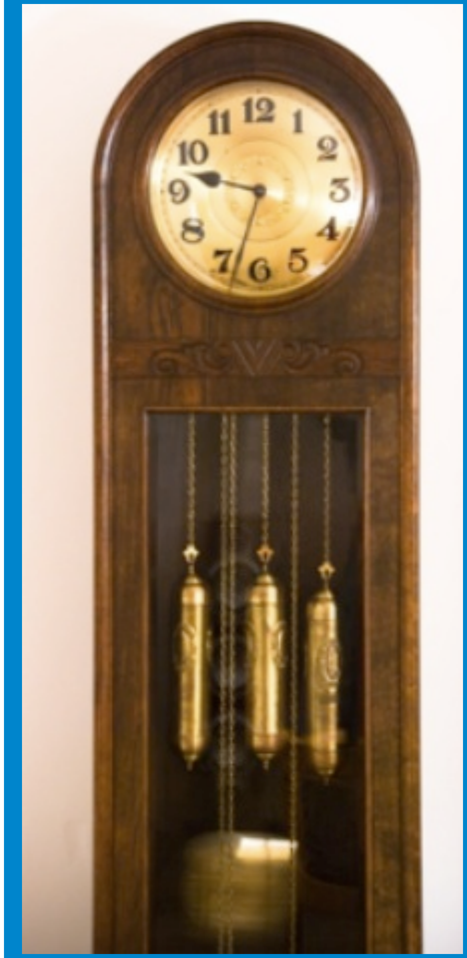
A mass attached to a spring oscillates as indicated in the graph below. At the time labeled by point P, the mass has:




1) positive velocity and positive acceleration.
2) positive velocity and negative acceleration.
3) negative velocity and positive acceleration.
4) negative velocity and negative acceleration.

# Things that Oscillate: II

pendulum



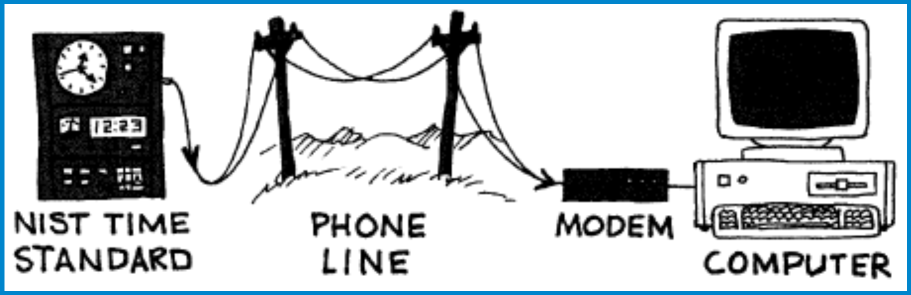


We use natural oscillations to measure time

1. pendulum
2. quartz crystals

Currently, we define “1 second” based on oscillations inside a Cesium atom:

**1 second = 9,192,631,770 oscillations**

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A grandfather clock pendulum with period of 1s in the classroom is placed on an elevator that is accelerating *downward* at 2.5 m/s$^2$. How will the clock's period in the elevator $T_{\text{elevator}}$ compare to its period in the classroom $T_{\text{classroom}}$?

1) $T_{\text{elevator}} = T_{\text{classroom}}$
2) $T_{\text{elevator}} < T_{\text{classroom}}$
3) $T_{\text{elevator}} > T_{\text{classroom}}$

# How long is the rope ($g$=32.2 ft/$s^2$)?

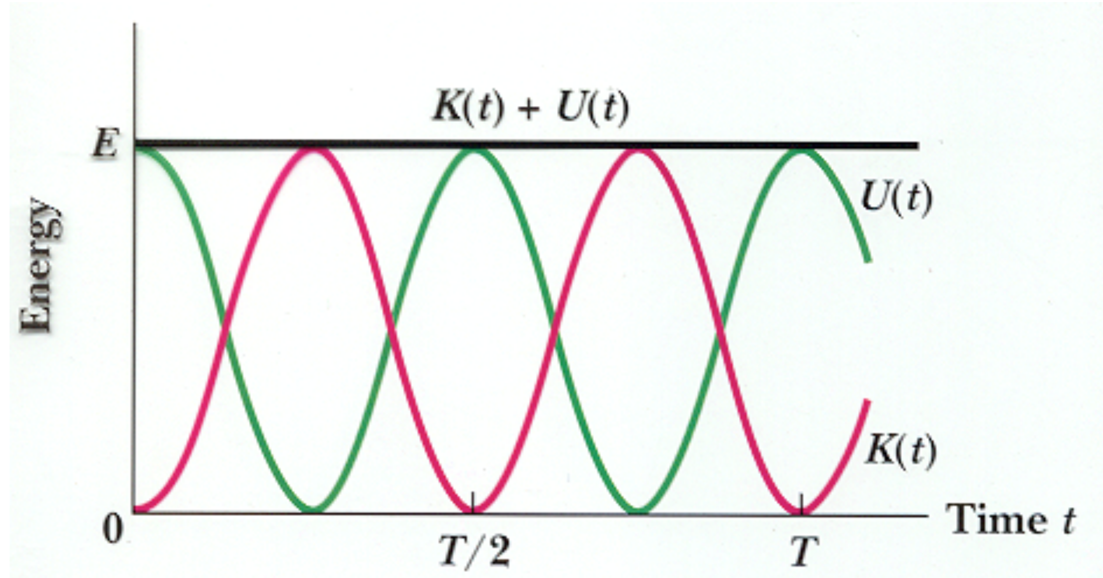
A: 7 ft
B: 12 ft
C: 17 ft
D: 22 ft
E: 27 ft
F: 32 ft

# Energy in SHM

The linear restoring force has an associated potential energy $U$ that scales as the square of the displacement. The mechanical energy

$$E_{mec} = U(t) + K(t)$$

remains constant if there is no friction (or *damping*).

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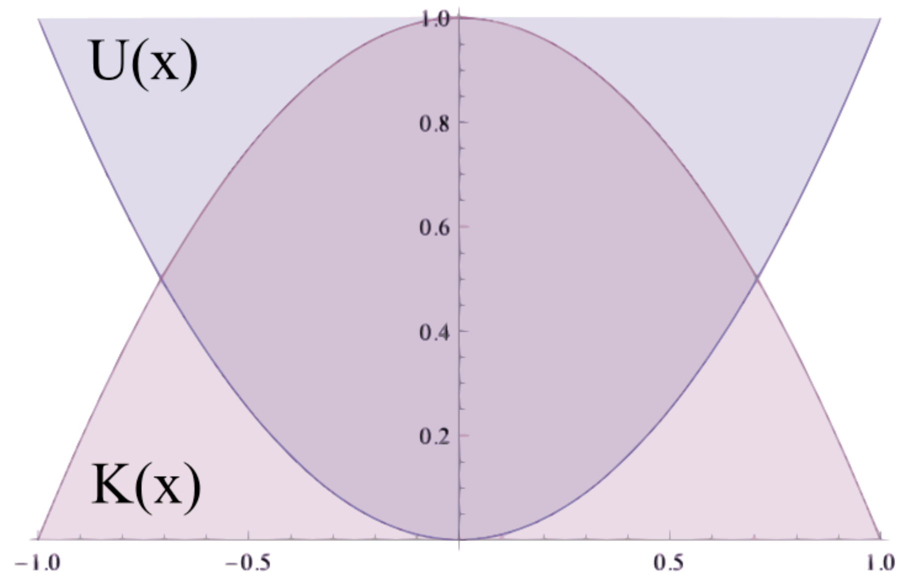
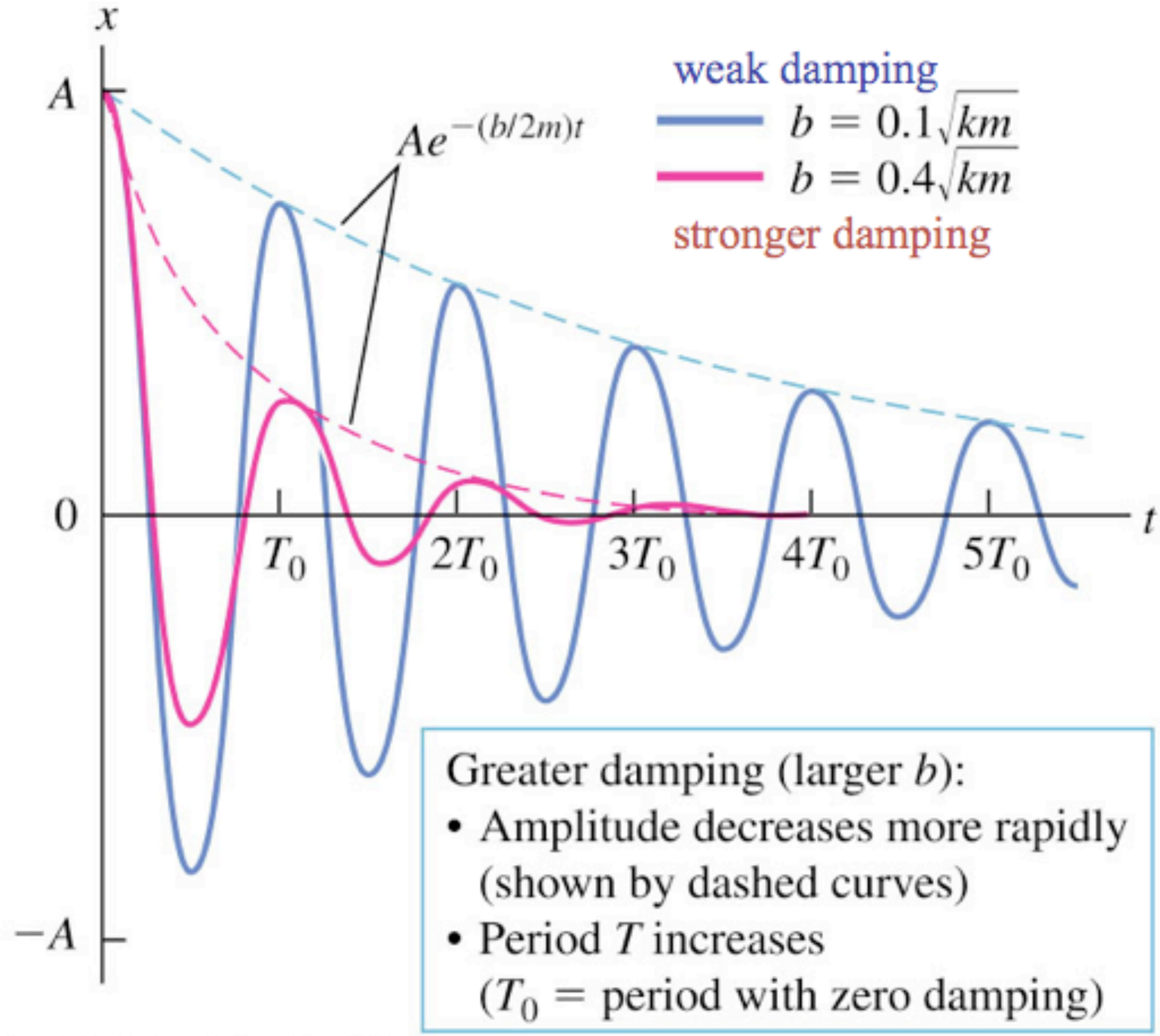
The kinetic and potential energies in SHM, shown as a function of displacement $x$, trade roles over the course of a cycle. The peak of one is the valley of the other.



# Damped harmonic motion:



# Damped oscillations

Friction or other sources of external work can lead to a ***loss of energy,*** (known as dissipation), from an oscillating system. This phenomenon is referred to as ***damping***.

Damping has two principal effects on the oscillating system. It

**- decreases the amplitude of the oscillations and**

**- decreases the frequency (increases the period) of oscillations.**

Damping introduces a separate timescale $T_{\mathrm{damping}}$ into the system. When compared to the oscillation period $T$, two regimes result

$T_{\mathrm{damp}} > T$, **slow energy loss, or *underdamped*,**

$T_{\mathrm{damp}} < T$, **rapid energy loss, or *overdamped*.**

# Natural frequency, driven oscillations, and resonance

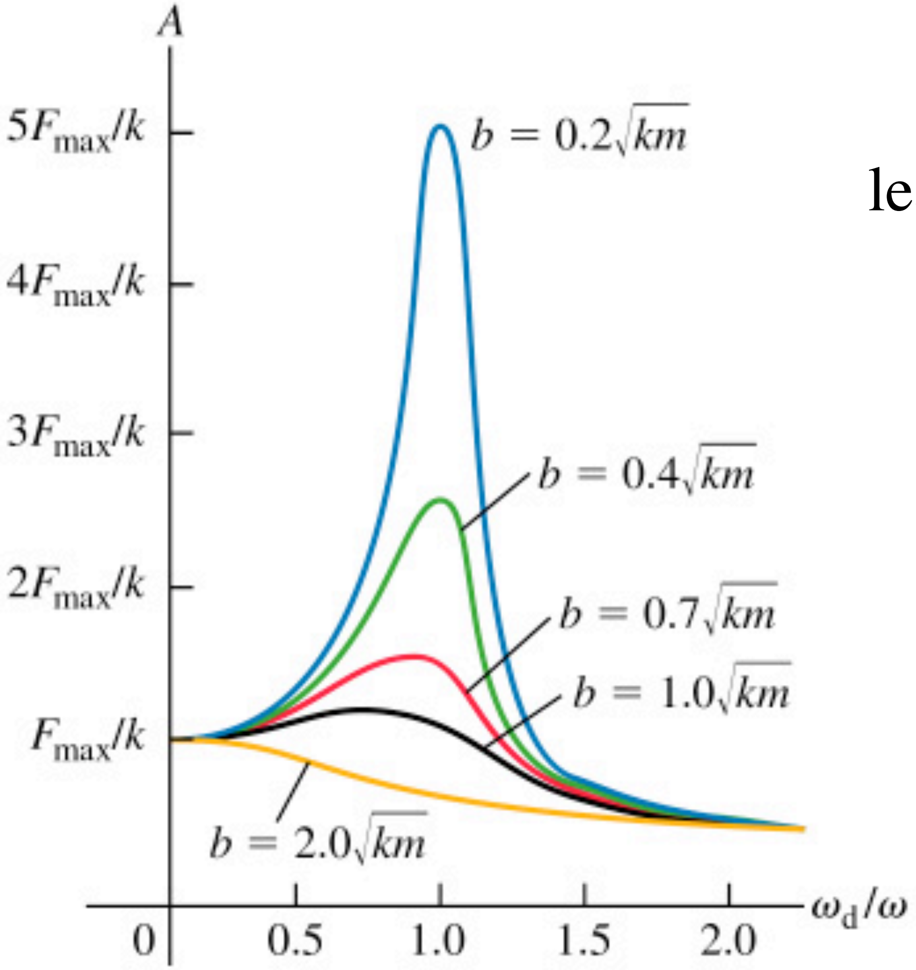
The oscillation frequency $f$ of a system undergoing simple harmonic motion (e.g., spring+mass or pendulum) is said to be that system's ***natural frequency***.

If we apply an ***external, oscillating force*** that serves to "drive" the system at some driving frequency $f_d$, then the system is able to absorb energy via the work done by the driving force.

The condition known as ***resonance*** is associated with the state that maximizes the efficiency of energy transfer from the driving force to the system. Resonance occurs ***when the driving frequency matches the system's natural frequency***

$$f_d = f$$

# Amplitude of a driven, damped spring-mass system:




driving force

$$F(t) = F_{max}\cos(\omega_d t)$$

leads to oscillation amplitude

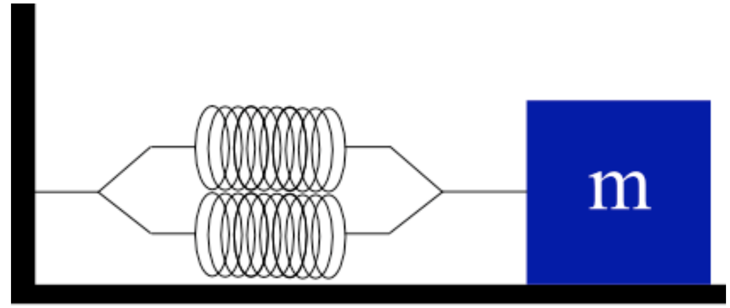
$$A = \frac{F_{max}}{\sqrt{(k - m\omega_d^2)^2 - b^2\omega_d^2}}$$

Greater damping (larger $b$):
- Peak becomes broader
- Peak becomes less sharp
- Peak shifts toward lower frequencies

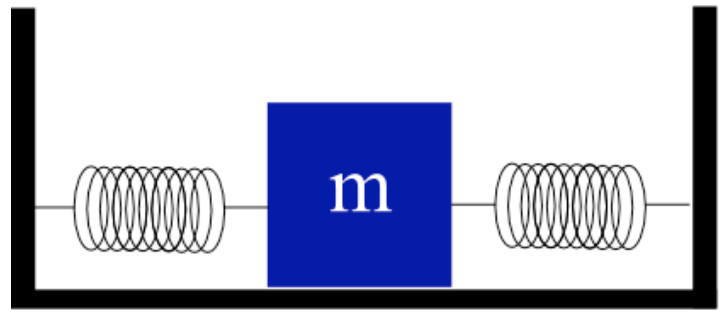
If $b > \sqrt{2km}$, peak disappears completely

Identical cubes of mass *m* on frictionless horizontal surfaces are attached to two springs, with spring constants $k_1$ and $k_2$, in the three cases shown at right. What is the relationship between their periods of oscillation?

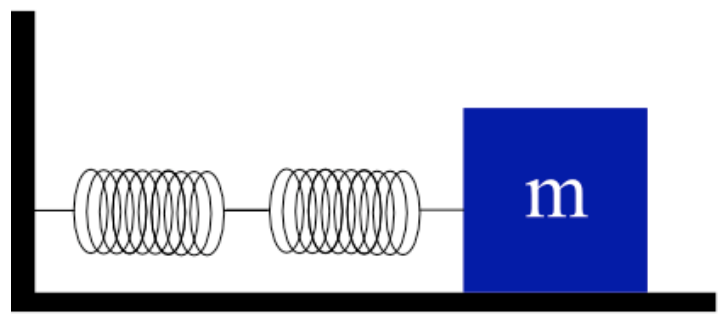
1) $T_a < T_b < T_c$
2) $T_a = T_b < T_c$
3) $T_a > T_b < T_c$
4) $T_a = T_b = T_c$
5) $T_a < T_b = T_c$



A



B



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