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Live from Ann Arbor, it's Tuesday Morning!

Ch 13 topics:

- restoring forces produce oscillations
- simple harmonic motion (SHM)
- damped harmonic motion
- natural frequency, driven oscillations and resonance

Midterm exam #3 is next Thursday, 29 November
covers chapters 9-12 (rotation through gravity)
bring **three** 3x5 notecards, calculator, #2 pencils

- Review on Monday, 26 Nov, 8:00-9:30pm

Which is your favorite FM station?

A: 88.1

B: 88.3

C: 88.7

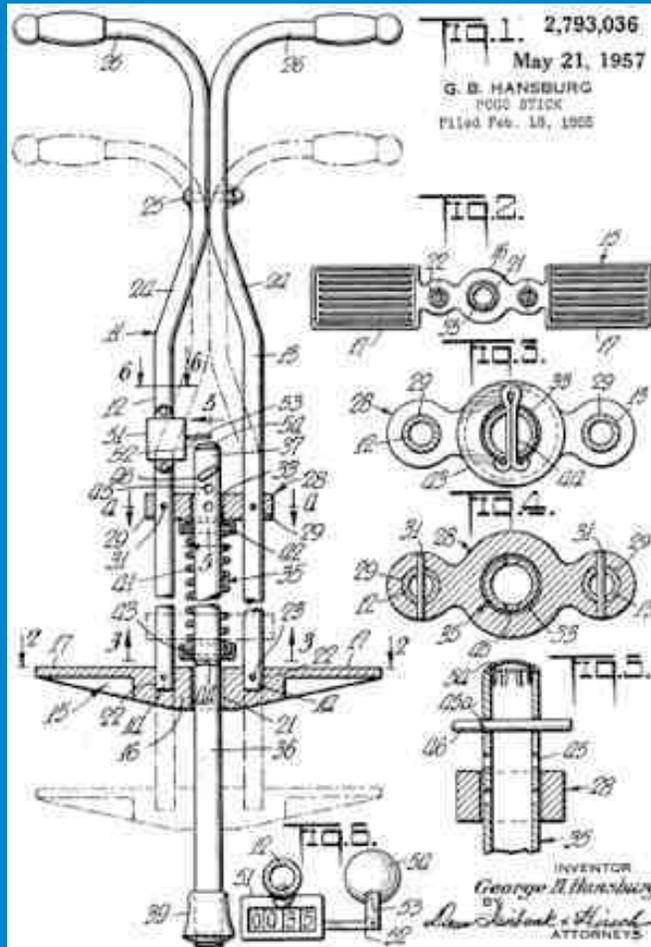
D: 95.5

E: 97.9

F:107.1

Things that Oscillate

mass on spring



Source: US Patent

pendulum



e⁻ in antenna

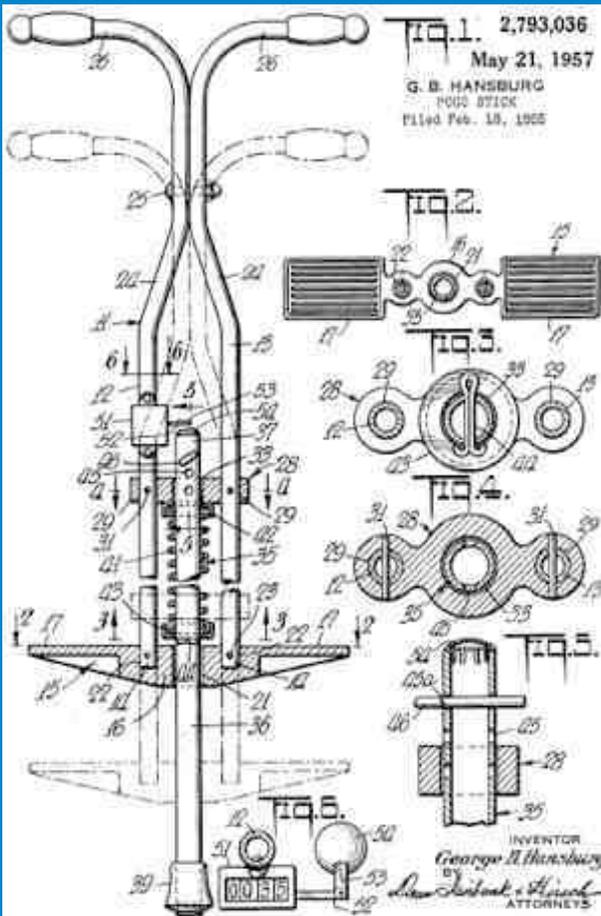


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Things that Oscillate: I

masses on springs



Source: US Patent



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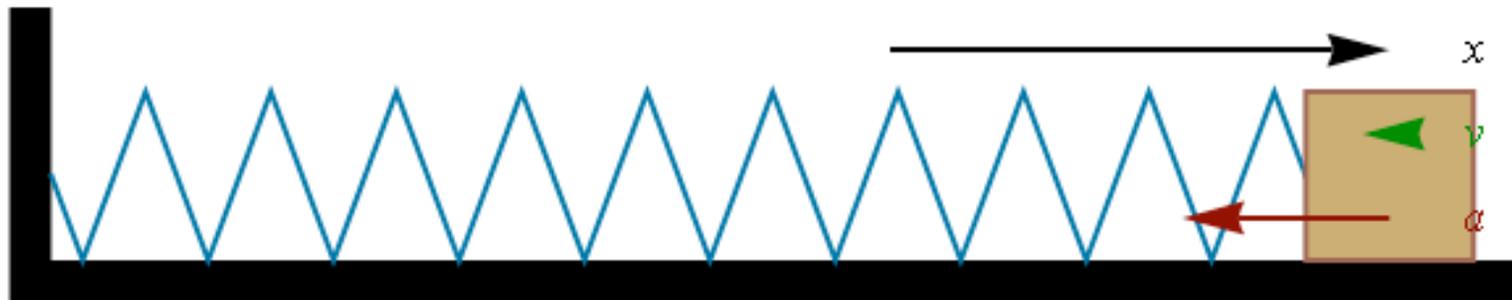
Source: Undetermined



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A *restoring force* leads to oscillations about a point of equilibrium.

<http://en.wikipedia.org/wiki/File:Muelle.gif>



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Linear Restoring Forces and Simple Harmonic Motion

A *linear restoring force* tends to push a system back toward a point of stable equilibrium, with a magnitude that varies linearly with the displacement away from equilibrium. An example is Hooke's law for an ideal spring

$$F = -kx$$

Applying Newton's second law gives a second-order ordinary differential equation

$$\frac{d^2 x}{dt^2} = -\frac{k}{m} x$$

the solution of which is a *sinusoidal variation of position* in time

$$x(t) = x_m \cos(\omega t + \phi) \quad (\omega = \sqrt{k/m})$$

Any system with displacement following this form is said to be undergoing *simple harmonic motion (SHM)*.

Conditions for SHM

Any system for which the acceleration varies with the negative of the displacement will exhibit SHM. The coefficient between a and x defines the square of the angular frequency ω^2 .

$$a(x) = -\omega^2 x \quad \longleftrightarrow \quad x(t) = x_m \cos(\omega t + \phi)$$

Descriptive features of SHM

Although the causes of SHM will vary from one system to another, the sinusoidal variation is a common element. All solutions are directly characterized by three features:

x_m : **maximum displacement amplitude** (or *amplitude*)

ω : **angular frequency**

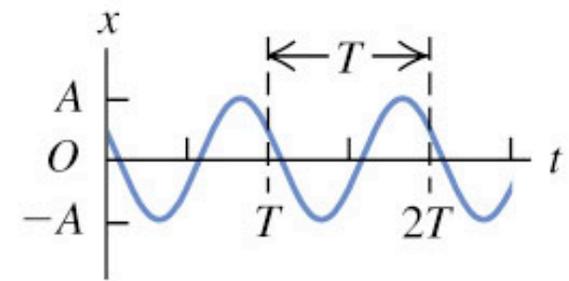
ϕ : **phase constant** (or *phase angle*)

and ω can alternately be specified by either of the following:

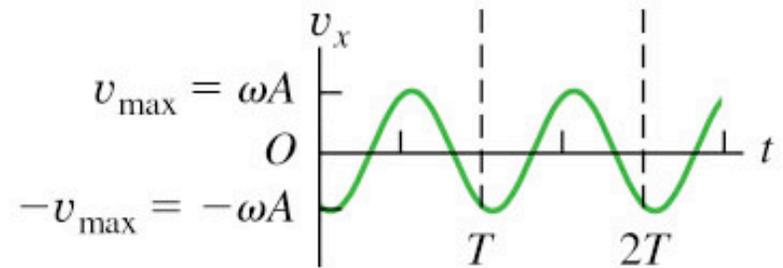
f : **frequency**, $f = \omega / 2\pi$ measured in Hertz ($1 \text{ Hz} = 1\text{s}^{-1}$)

T : **period**, $T = 1/f = 2\pi/\omega$

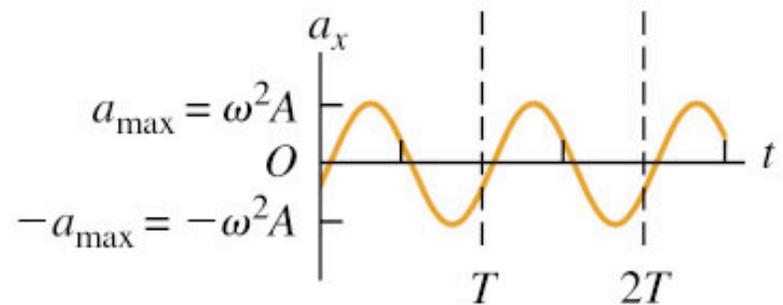
The behavior of *simple harmonic motion* is the same as a linear projection of circular motion.



(a) Displacement

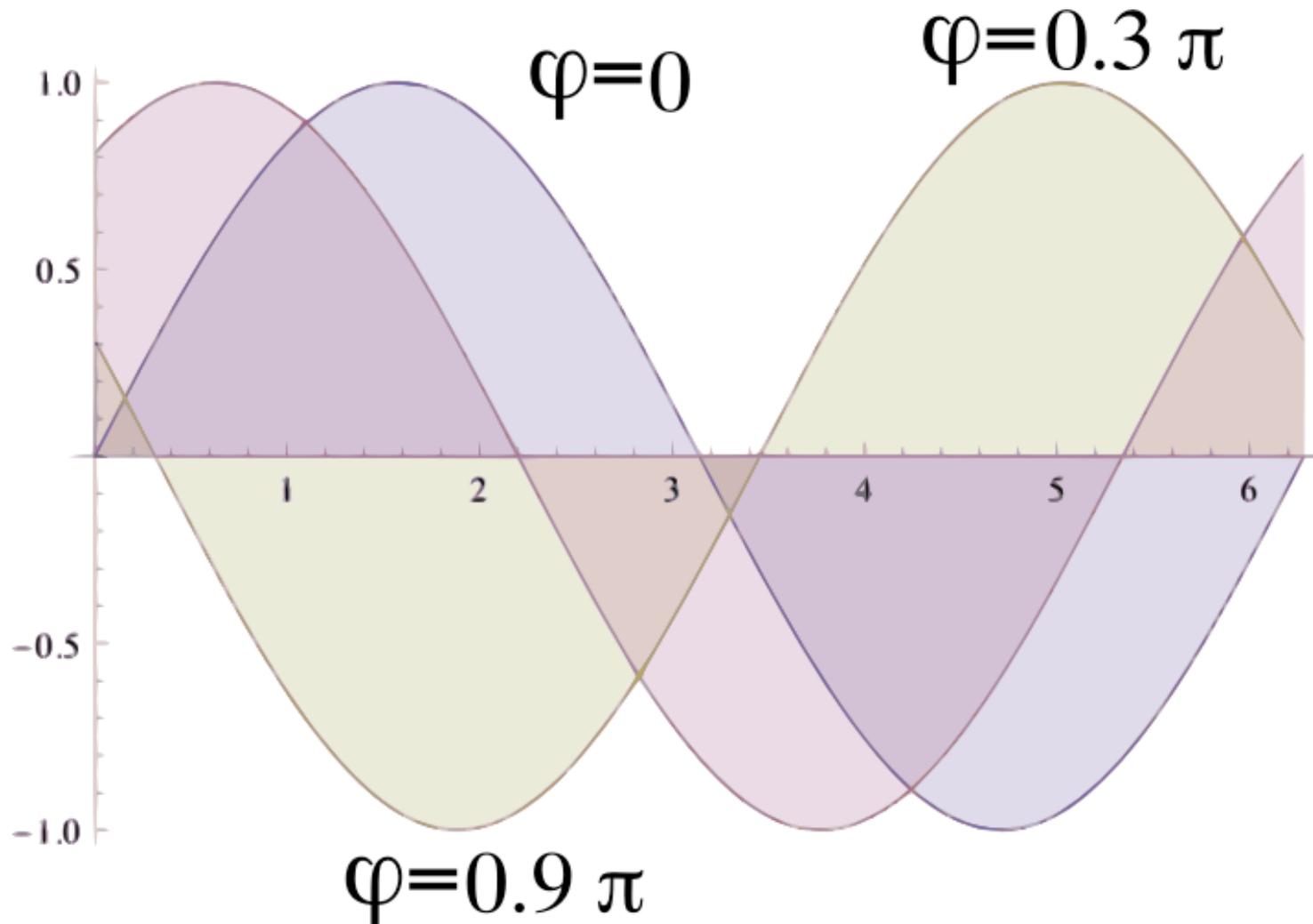


(b) Velocity



(c) Acceleration

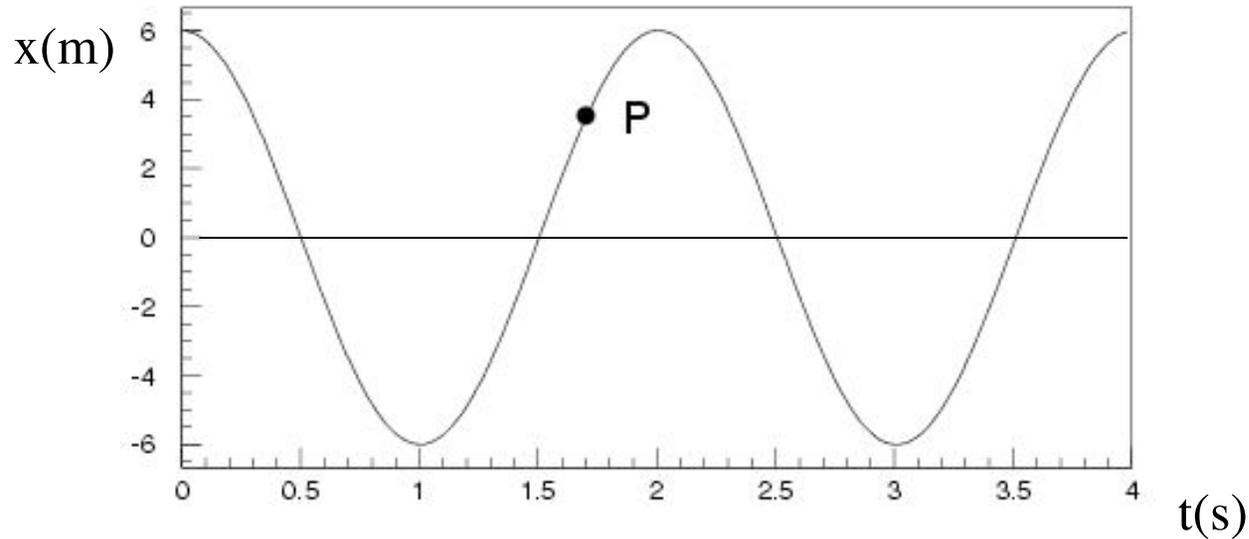
[http://en.wikipedia.org/wiki/
File:Simple_Harmonic_Motion_Or
bit.gif](http://en.wikipedia.org/wiki/File:Simple_Harmonic_Motion_Orbit.gif)



$$x(t) = x_m \cos(\omega t + \phi)$$

The behavior at $t = 0$ defines the *phase constant* ϕ .

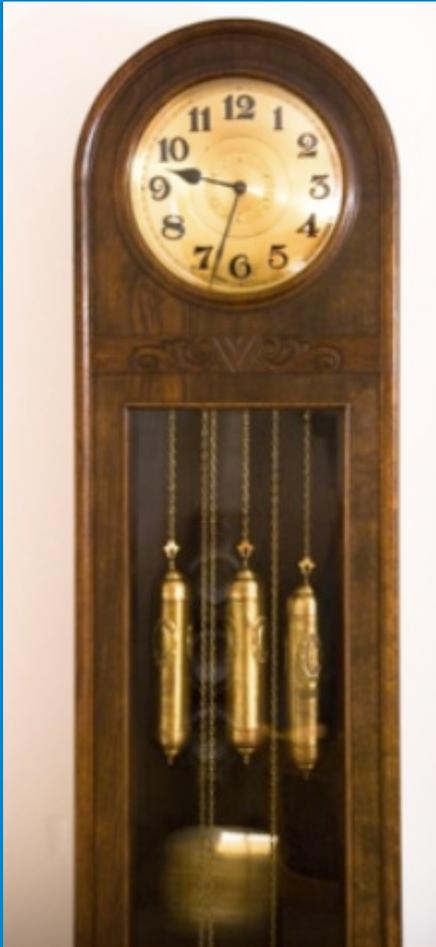
A mass attached to a spring oscillates as indicated in the graph below. At the time labeled by point P, the mass has:



- 1) positive velocity and positive acceleration.
- 2) positive velocity and negative acceleration.
- 3) negative velocity and positive acceleration.
- 4) negative velocity and negative acceleration.

Things that Oscillate: II

pendulum



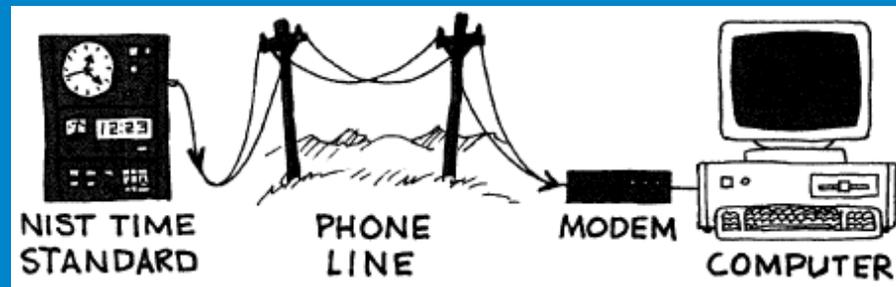
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We use natural oscillations to measure time

1. pendulum
2. quartz crystals

Currently, we define “1 second” based on oscillations inside a Cesium atom:

1 second = 9,192,631,770 oscillations



(303) 499-7111 :: <http://tf.nist.gov/>

A grandfather clock pendulum with period of 1s in the classroom is placed on an elevator that is accelerating *downward* at 2.5 m/s^2 . How will the clock's period in the elevator T_{elevator} compare to its period in the classroom $T_{\text{classroom}}$?

1) $T_{\text{elevator}} = T_{\text{classroom}}$

2) $T_{\text{elevator}} < T_{\text{classroom}}$

 3) $T_{\text{elevator}} > T_{\text{classroom}}$

How long is the rope ($g=32.2 \text{ ft/s}^2$)?

A: 7 ft

B: 12 ft

C: 17 ft

D: 22 ft

E: 27 ft

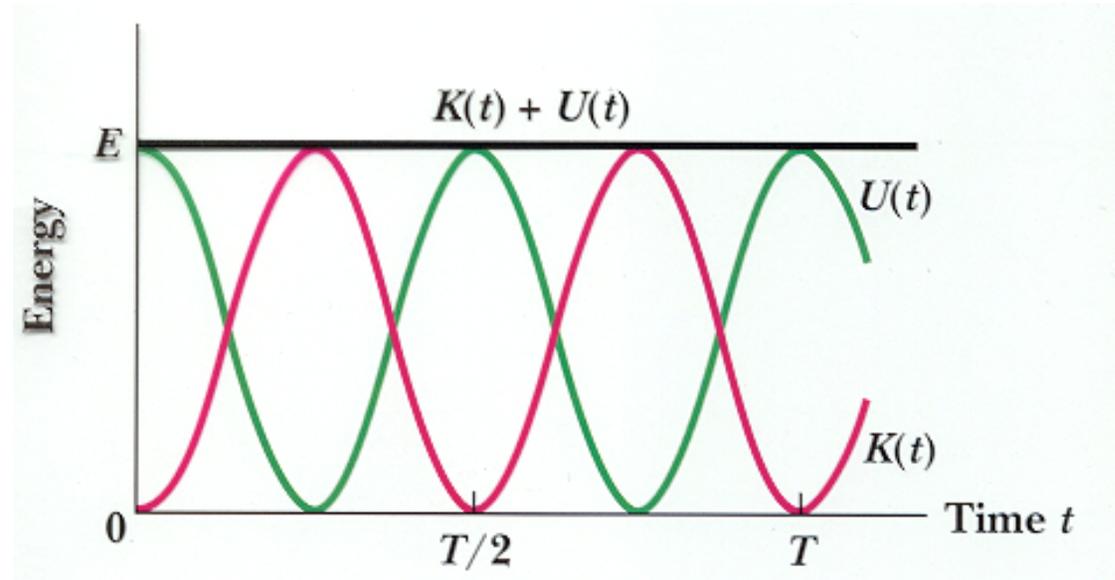
F: 32 ft

Energy in SHM

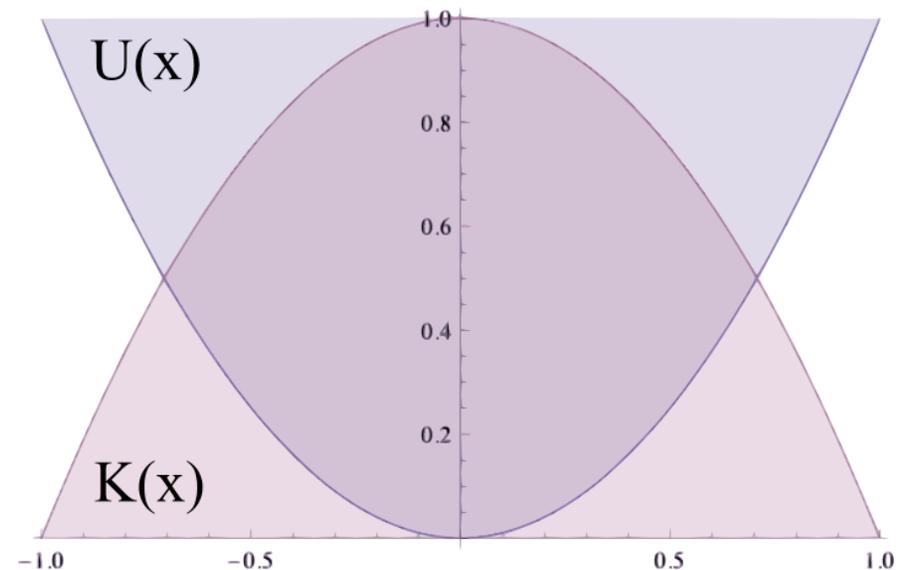
The linear restoring force has an associated potential energy U that scales as the square of the displacement. The mechanical energy

$$E_{\text{mec}} = U(t) + K(t)$$

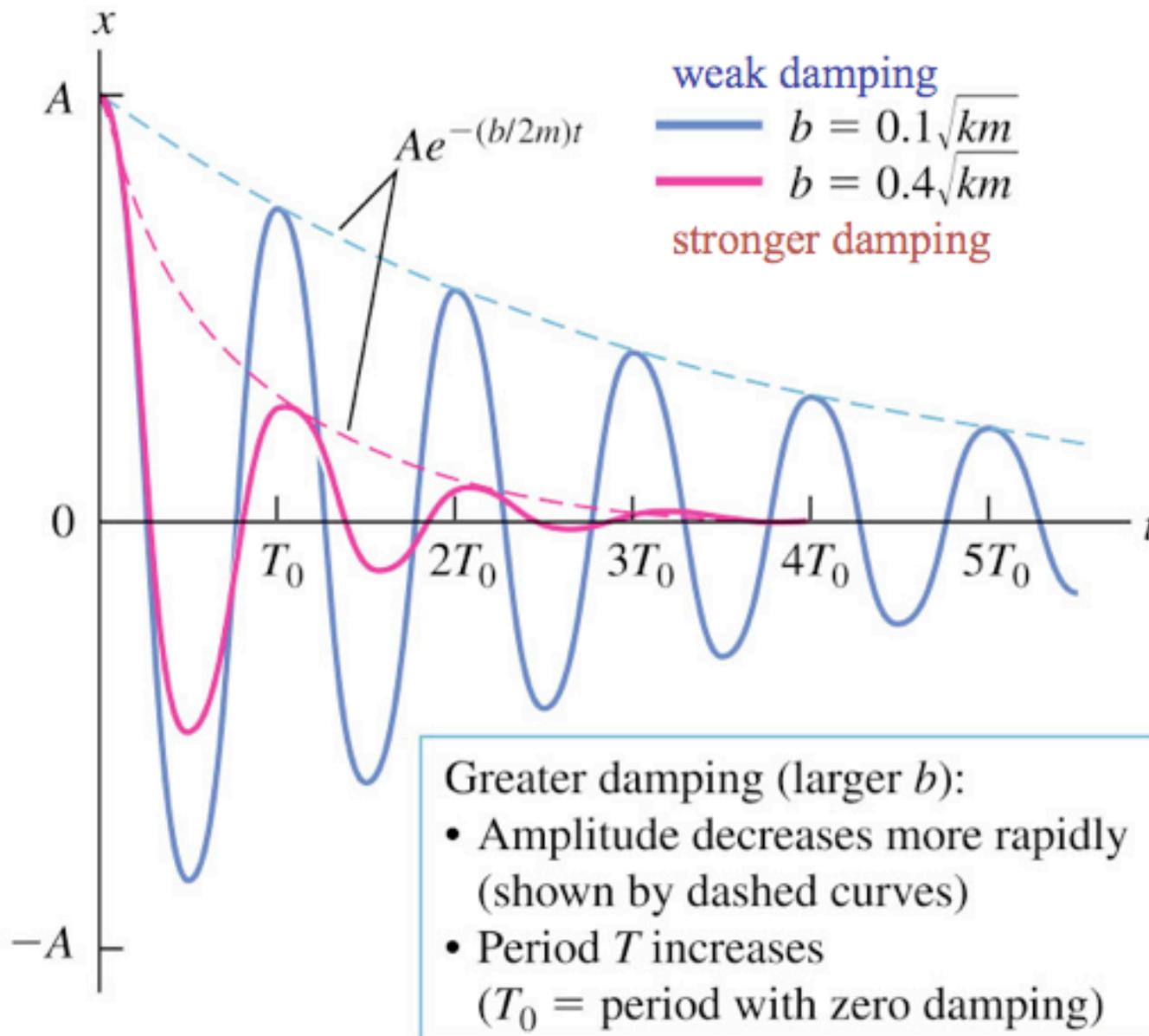
remains constant if there is no friction (or *damping*).



The kinetic and potential energies in SHM, shown as a function of displacement x , trade roles over the course of a cycle. The peak of one is the valley of the other.



Damped harmonic motion:



Damped oscillations

Friction or other sources of external work can lead to a *loss of energy*, (known as *dissipation*), from an oscillating system. This phenomenon is referred to as *damping*.

Damping has two principal effects on the oscillating system. It

- **decreases the amplitude of the oscillations and**
- **decreases the frequency (increases the period) of oscillations.**

Damping introduces a separate timescale T_{damping} into the system. When compared to the oscillation period T , two regimes result

$T_{\text{damp}} > T$, slow energy loss, or *underdamped*,

$T_{\text{damp}} < T$, rapid energy loss, or *overdamped*.

Natural frequency, driven oscillations, and resonance

The oscillation frequency f of a system undergoing simple harmonic motion (e.g., spring+mass or pendulum) is said to be that system's *natural frequency*.

If we apply an *external, oscillating force* that serves to “drive” the system at some driving frequency f_d , then the system is able to absorb energy via the work done by the driving force.

The condition known as resonance is associated with the state that maximizes the efficiency of energy transfer from the driving force to the system. Resonance occurs *when the driving frequency matches the system's natural frequency*

$$f_d = f$$

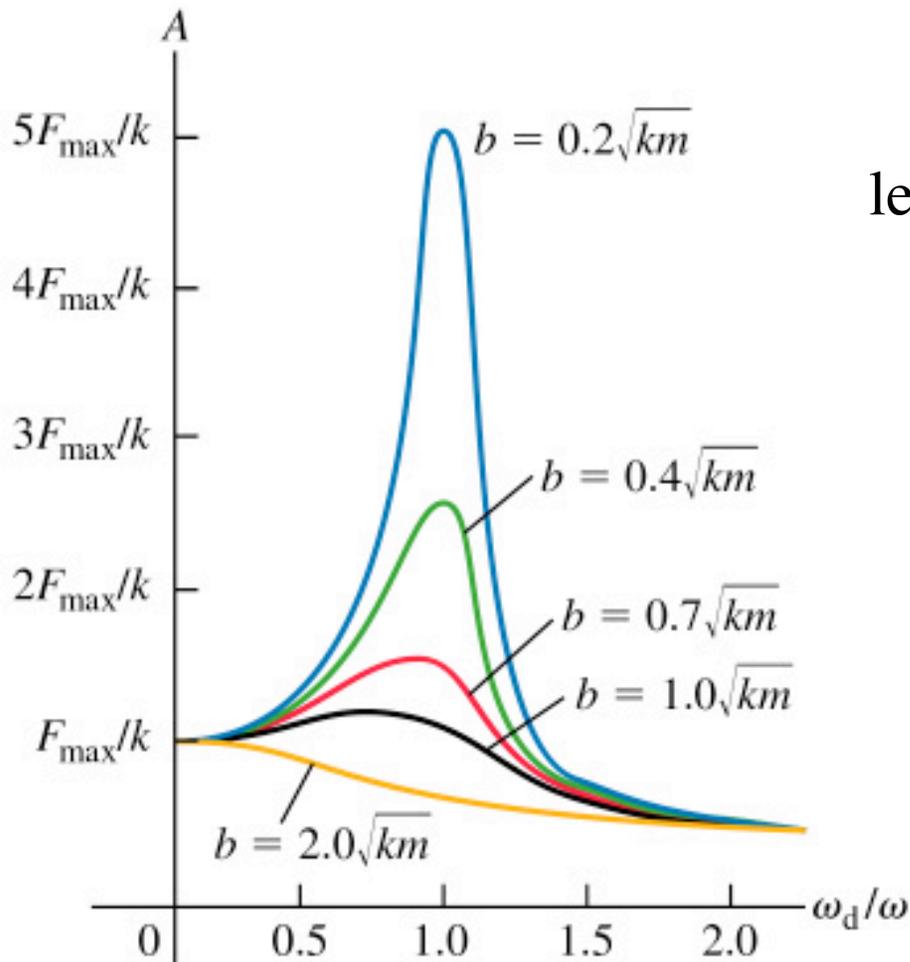
Amplitude of a driven, damped spring-mass system:

driving force

$$F(t) = F_{\max} \cos(\omega_d t)$$

leads to oscillation amplitude

$$A = \frac{F_{\max}}{\sqrt{(k - m\omega_d^2)^2 - b^2\omega_d^2}}$$



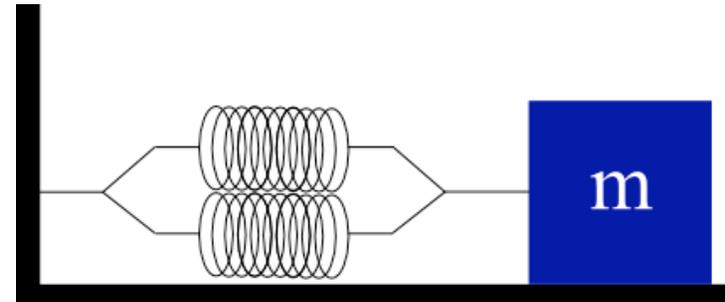
Greater damping (larger b):

- Peak becomes broader
- Peak becomes less sharp
- Peak shifts toward lower frequencies

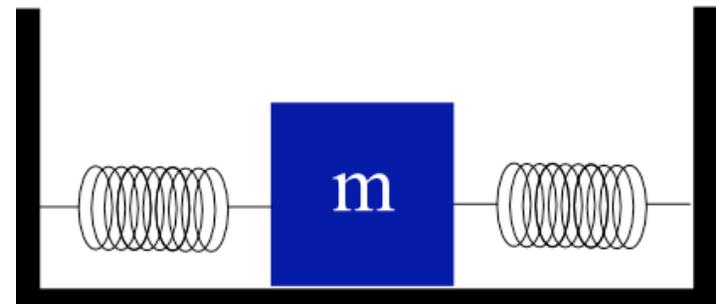
If $b > \sqrt{2km}$, peak disappears completely

Identical cubes of mass m on frictionless horizontal surfaces are attached to two springs, with spring constants k_1 and k_2 , in the three cases shown at right. What is the relationship between their periods of oscillation?

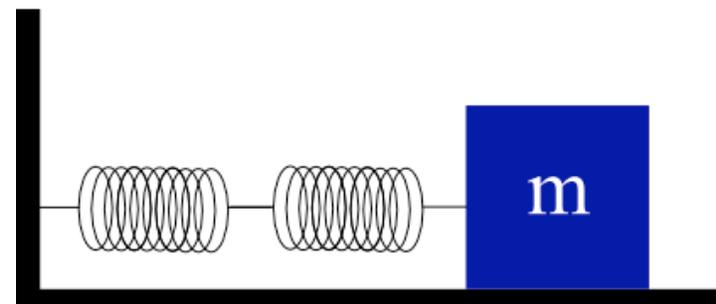
- 1) $T_a < T_b < T_c$
 2) $T_a = T_b < T_c$
 3) $T_a > T_b < T_c$
 4) $T_a = T_b = T_c$
 5) $T_a < T_b = T_c$



A



B



C